Communications in Combinatorics and Optimization
Vol. xx, No. x (xxxx), pp. 1-16
https://doi.org/10.22049/cco.2026.31466.2861

CCO
Commun. Comb. Optim.

Research Article

A feasible predictor-corrector interior-point method for monotone weighted linear complementarity problems

Behrouz Kheirfam[1,*], Imelda S. Aniversario[2], Afsaneh Nasrollahi[1]

[1]Department of Applied Mathematics, Azarbaijan Shahid Madani University, Tabriz, I.R. Iran
*b.kheirfam@azaruniv.ac.ir

[2]Center for Mathematical and Theoretical Physical Sciences PRISM,
MSU-Iligan Institute of Technology, Iligan City, Philippines
imelda.aniversario@g.msuiit.edu.ph

Received: 16 November 2025; Accepted: 14 March 2026
Published Online: 17 April 2026

Abstract: This work presents a predictor-corrector interior-point algorithm for solving the weighted linear complementarity problem. By applying Newton's type method to the central path system, the search directions are obtained. The algorithm works in the τ-neighborhood, which measures the proximity of iterates to the central path. By suitable choice of parameters, the global convergence of the method under mild conditions is guaranteed. The iteration bound derived to find ε-approximate solution matches the best known iteration bound for these types of problems. To the best of our knowledge, this is the first work based on these types of search directions.

Keywords: Weighted linear complementarity problem, Interior-point methods, Predictor-corrector methods, Polynomial complexity.

AMS Subject classification: 90C51, 90C33

1. Introduction

The goal of the weighted linear complementarity problem (WLCP) is to find the vector $(x, s, y) \in \mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R}^m$ belonging to the intersection of a manifold with nonnegative orthant such that the Hadamard product of the vectors x and s equals a given weight vector $w \geq 0$. That is,

$$\begin{aligned} Px + Qs + Ry = a, \quad x, s \geq 0 \\ xs = w, \end{aligned} \tag{1.1}$$

* *Corresponding Author*

where $P, Q \in \mathbf{R}^{(n+m)\times n}, R \in \mathbf{R}^{(n+m)\times m}$ are given matrices, $a \in \mathbf{R}^{n+m}$ is a given vector, and $w \in \mathbf{R}^n_+$ is a given weight vector. The importance of WLCP is that a large class of problems in science, engineering and economics can be formulated as a WLCP. Potra [16] proposed a generalization of the linear programming and weighted centering problem [2], called the quadratic programming and weighted centering problem, and showed that this problem and its dual can be formulated as a monotone WLCP. He also analyzed two interior-point methods (IPMs) for solving the general monotone WLCP. The first method is an extension of the large-step path-following method of McShane [13], which requires one matrix factorization per iteration. The second method is an extension of the Mizuno-Todd-Ye (MTY) predictor-corrector algorithm [14], which requires two matrix factorizations at each iteration. In general the corrector-predictor IPMs are based only on one neighborhood of the central path. The main purpose of the corrector is to increase the proximity to the central path, while the aim of the predictor step is to obtain a point on the boundary of the neighborhood. In [17], Potra extended the concept of sufficiency introduced by Cottle et al. [4] for LCP to WLCP and then proposed a corrector-predictor IPM to find its ε-approximate solution. Furthetmore, he showed that a strictly feasible sufficient WLCP is solvable. Tang and Zhou [20] presented a damped Gauss-Newton method for solving non-monotone WLCP based on derivative-free non-monotone linear search and proved that their method is globally convergent without any problem assumptions.

Full Newton-step IPMs were first proposed for solving linear optimization (LO) by Roos et al. [19] and have the advantage that no line-search is required. Various full-Newton step algorithms have been presented in the literature to solve various types of optimization problems, see for example [1, 5, 21–23]. Asadi et al. [3] extended the full-Newton step IPM to the monotone WLCP and established the feasibility of the full steps and a quadratic rate of convergence to the target points on the central path. Recently, Kheirfam [12] proposed a full-Newton step IPM for WLCP based on the algebraic equivalent transformation (AET) introduced in [5]. Kheirfam [9–11] proposed corrector-predictor IPMs for $P_*(\kappa)$-horizontal LCP, semidefinite optimization (SDO) and convex quadratic symmetric cone optimization (CQSCO), respectively. Potra [15] proposed an IPM for monotone LCPs which first performs a corrector step if initial interior-point is not well centered and after that a predictor step. Potra and Liu [18] proposed an IPM for sufficient LCPs which uses a wide neighborhood of the central path and the algorithm does not depend on the handicap of the problem.

Darvay et al. [6] introduced a corrector-predictor IPM for LO based on the AET approach using the difference of identity function and square root function. Later on, Darvay et al. [7] extended the corrector-predictor IPM given in Darvay et al. [6] to $P_*(\kappa)$-LCP. Recently, Darvay et al. [8] proposed a corrector-predictor IPM for solving $P_*(\kappa)$-LCP which uses the new type of AET on the centring equations of the system defining the central path.

The main goal of this paper is to present a predictor-corrector IPM for solving WLCP. The algorithm is based on Newton's directions obtained from the central path system and works in the τ-neighborhood of the central path. We present the complexity analysis of the proposed predictor-corrector algorithm and derive the iteration bound. When the weight vector is zero, WLCP (1.1) includes the general linear complementarity probelm (LCP) studied by Ye [24] which contains the standard LCP [10] as a special case. However, the analysis for the predictor steps is more complicated than the ones done for the predictor-corrector algorithms for other problems addressed in [9–11] mainly because of the nonzero weight vector. Numerical results show that the proposed algorithm performs well on a small set of test problems.

The paper is organized as follows. In Section 2, we briefly introduce concepts such as central path system and search directions. In Section 3, we analyze the corrector step and the predictor step. Then, the framework of predictor-corrector algorithm for WLCP is described. The complexity of the proposed algorithm is derived in Section 4. Some numerical results are given in Section 5. Finally, we provide concluding remarks in Section 6.

2. The Central Path

Throughout this paper we assume that the matrix R has full column rank and the WLCP (1.1) is monotone, in the sense that:

$$P\Delta x + Q\Delta s + R\Delta y = 0 \ \text{ implies } \ \Delta x^T \Delta s \geq 0.$$

Assume that

$$\mathcal{F} := \{(x, s, y) \in \mathbf{R}^n_+ \times \mathbf{R}^n_+ \times \mathbf{R}^m : Px + Qs + Ry = a\}$$

denotes the set of feasible solutions, and

$$\mathcal{F}^* := \{(x, s, y) \in \mathcal{F} : \ xs = w\}$$

denotes the solution set of (1.1). Moreover, the set of all strictly feasible points of (1.1) is given by

$$\mathcal{F}^0 := \{(x, s, y) \in \mathcal{F} : \ x, \ s > 0\}.$$

We consider the same central path for WLCP as introduced in [16, 17]. For a strictly feasible starting point $(x^0, s^0, y^0) \in \mathcal{F}^0$, we denote

$$\mu^0 = \tfrac{(x^0)^T s^0}{n}, \quad c = x^0 s^0, \quad \gamma = \tfrac{\min c}{\mu^0}, \quad w(\mu) = (1 - \tfrac{\mu}{\mu^0})w + \tfrac{\mu}{\mu^0} c, \tag{2.1}$$

where $\mu \in (0, \mu^0]$ and assume that $c \neq w$. The central path of the WLCP given as (1.1), determined by (x^0, s^0, y^0), is the set of all points $(\mu; x, s, y)$, with $\mu \in (0, \mu^0]$, satisfying

$$\begin{aligned} Px + Qs + Ry = a, \qquad x, s > 0 \\ xs = w(\mu). \end{aligned} \tag{2.2}$$

Under the assumption that the WLCP is monotone and $\mathcal{F}^0 \neq \emptyset$, it is proved that system (2.2) has a unique solution for any $\mu \in (0, \mu^0]$ ([17, Proposition 3.1]). Moreover, by construction, $(\mu^0; x^0, s^0, y^0)$ belongs to the central path (see [12]), and if $\mu \downarrow 0$ then the central path converges to a solution of (1.1). Application of Newton's type method to system (2.2) with fixed μ, with the assumption that $(x, s, y) \in \mathcal{F}^0$, produces the following system for search directions $\Delta x, \Delta s$, and Δy:

$$\begin{aligned} P\Delta x + Q\Delta s + R\Delta y &= 0, \\ s\Delta x + x\Delta s &= w(\mu) - xs. \end{aligned} \tag{2.3}$$

Consider the following notations:

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v\Delta x}{x}, \quad d_s = \frac{v\Delta s}{s}.$$

One easily verifies that the system (2.3) can be written as

$$\begin{aligned} \tilde{P}d_x + \tilde{Q}d_s + R\Delta y &= 0, \\ d_x + d_s &= v^{-1}\tfrac{w(\mu)}{\mu} - v, \end{aligned}$$

where $\tilde{P} = PV^{-1}X,\ \tilde{Q} = QV^{-1}S,\ V = \text{diag}(v),\ X = \text{diag}(x)$ and $S = \text{diag}(s)$. We introduce the proximity of a point (x, s, y) to the central path (2.2) by

$$\delta(x, s; \mu) = \left\| \frac{w(\mu)}{\mu} - v^2 \right\|.$$

Note that for any $(x, s, y) \in \mathcal{F}$, we have

$$xs = w(\mu) \Leftrightarrow v^2 = \frac{w(\mu)}{\mu} \Leftrightarrow \delta(x, s; \mu) = 0.$$

Hence, the value $\delta(x, s; \mu)$ can be considered as a proximity measure for the distance between the given triple (x, s, y) and the central path (2.2).
Now, using the definition of $\delta(x, s; \mu)$, we define the τ-neighborhood of the central path in the following way:

$$\mathcal{N}_2(\tau, \mu) := \{(x, s, y) \in \mathcal{F}^0 : \ \delta(x, s; \mu) \leq \tau\} = \{(x, s, y) \in \mathcal{F}^0 : \ \left\|\frac{w(\mu)}{\mu} - v^2\right\| \leq \tau\},$$

where τ is threshold parameter and $\mu > 0$ is fixed.
The following lemma is essential in the analysis of Algorithm 1.

Lemma 1. *If $\delta := \delta(x, s; \mu) \leq \gamma$, then for each $i = 1, \ldots, n$, we have*

$$v_i \geq \sqrt{\gamma - \delta},$$

where γ is defined in (2.1).

Proof From the definition of δ, we have

$$\delta(x, s; \mu) = \left\| \frac{w(\mu)}{\mu} - v^2 \right\| \geq \left| \frac{w_i(\mu)}{\mu} - v_i^2 \right|, \ i = 1, 2, \ldots, n.$$

This implies

$$v_i^2 \geq \frac{w_i(\mu)}{\mu} - \delta \geq \frac{c}{\mu^0} - \delta \geq \gamma - \delta,$$

where c and γ are defined in (2.1). Taking square roots proves the lemma. □

3. Predictor-Corrector Algorithm for WLCP

In this section, we present a predictor-corrector interior-point algorithm for solving WLCP.

3.1. The Corrector Step.

In a corrector step, we obtain d_x^c, d_s^c and $\Delta^c y$ by solving the scaled system

$$\begin{aligned} \tilde{P} d_x^c + \tilde{Q} d_s^c + R\Delta^c y = 0, \\ d_x^c + d_s^c = v^{-1}\tfrac{w(\mu)}{\mu} - v. \end{aligned} \tag{3.1}$$

Newton directions of the original system (2.3); i.e.,

$$\Delta^c x = \frac{x}{v} d_x^c, \qquad \Delta^c s = \frac{s}{v} d_s^c \tag{3.2}$$

can be easily computed and the corrector step is obtained by a full-Newton step as follows:

$$(x_+, s_+, y_+) = (x, s, y) + (\Delta^c x, \Delta^c s, \Delta^c y).$$

The corrector step of the proposed IPM is similar to the small-update IPM introduced in [3]. Hence, for the analysis of this step results in [3] can be applied. The next lemma gives the strict feasibility of the iterates obtained by the corrector step.

Lemma 2. ([3, Lemma 4.5]) *If $\delta = \delta(x, s; \mu) \leq \frac{2\gamma}{1+\sqrt{1+\sqrt{2}}}$, then the full-Newton step is feasible. Moreover, we have*

$$\delta^+ := \delta(x_+, s_+; \mu) \leq \frac{\delta^2}{2\sqrt{2}(\gamma - \delta)}.$$

The following lemma gives an upper bound of the duality gap after a corrector step.

Lemma 3. ([3, Lemma 4.4]) *After a full-Newton step, the following inequality holds:*

$$x_+^T s_+ \leq e^T w(\mu) + \frac{\mu\delta^2}{2\sqrt{2}(\gamma-\delta)}.$$

The next lemma shows the local quadratic convergence of the corrector step.

Lemma 4. ([3, Corollary 4.3]) *If* $\delta \leq (1-\frac{1}{2\sqrt{2}})\gamma$, *then* $\delta^+ \leq \left(\frac{\delta}{\sqrt{\gamma}}\right)^2$.

3.2. The Predictor Step.

In the predictor step, we follow the search directions d_x^p, d_s^p and $\Delta^p y$ by solving the following system

$$\begin{aligned} \tilde{P}_+ d_x^p + \tilde{Q}_+ d_s^p + R\Delta^p y = 0, \\ d_x^p + d_s^p = v_+^{-1}\tfrac{w}{\mu} - v_+, \end{aligned} \tag{3.3}$$

where

$$v_+ = \sqrt{\frac{x_+ s_+}{\mu}}, \quad \tilde{P}_+ = PV_+^{-1}X_+, \quad \tilde{Q}_+ = QV_+^{-1}S_+, \quad V_+ = \mathrm{diag}(v_+)$$

and $X_+ = \mathrm{diag}(x_+),\ S_+ = \mathrm{diag}(s_+)$. It is easy to see that

$$\Delta^p x = \frac{x_+}{v_+} d_x^p, \qquad \Delta^p s = \frac{s_+}{v_+} d_s^p. \tag{3.4}$$

We define

$$x^p = x_+ + \theta\Delta^p x, \qquad s^p = s_+ + \theta\Delta^p s, \qquad y^p = y_+ + \theta\Delta^p y, \quad \mu^p = (1-\theta)\mu.$$

The step size along this direction is taken as

$$\bar{\theta} = \max\{\hat{\theta} \in [0,1] : (x^p, s^p, y^p) \in \mathcal{N}_2(\tau, \mu^p), \forall\ \theta \in [0, \hat{\theta}]\}. \tag{3.5}$$

In the next lemma, we present a sufficient condition that guarantees the strict feasibility of the predictor step.

Lemma 5. *Let* $x_+ > 0$ *and* $s_+ > 0$ *be feasible solution obtained after a corrector step and* $\mu > 0$. *Furthermore, let* $\delta^+ < \gamma$ *and* $\theta \in (0,1)$. *Let* x^p *and* s^p *be the iterates after a predictor step. Then* $x^p > 0$ *and* $s^p > 0$ *if*

$$h(\delta^+, \theta, \gamma) := \gamma - \delta^+ - \frac{\theta^2}{2\sqrt{2}(1-\theta)(\gamma-\delta^+)}\left(\delta^+ + \frac{1}{\mu^0}\|c-w\|\right)^2 > 0.$$

Proof For each $\alpha \in [0,1]$, we consider the following notations:

$$x^p(\alpha) = x_+ + \alpha\theta\Delta^p x = \frac{x_+}{v_+}(v_+ + \alpha\theta d_x^p),$$
$$s^p(\alpha) = s_+ + \alpha\theta\Delta^p s = \frac{s_+}{v_+}(v_+ + \alpha\theta d_s^p).$$

Using the second equation of the predictor scaled system (3.3), we have

$$\begin{aligned} x^p(\alpha)s^p(\alpha) &= \frac{x_+s_+}{v_+^2}(v_+ + \alpha\theta d_x^p)(v_+ + \alpha\theta d_s^p) \\ &= \frac{x_+s_+}{v_+^2}\left(v_+^2 + \alpha\theta v_+(d_x^p + d_s^p) + \alpha^2\theta^2 d_x^p d_s^p\right) \\ &= \mu\Big(v_+^2 + \alpha\theta\big(\frac{w}{\mu} - v_+^2\big) + \alpha^2\theta^2 d_x^p d_s^p\Big) \\ &= \mu\Big((1-\alpha\theta)v_+^2 + \alpha\theta\frac{w}{\mu} + \alpha^2\theta^2 d_x^p d_s^p\Big). \end{aligned} \tag{3.6}$$

From (3.6) we obtain the following inequality

$$\begin{aligned} \min\left(\frac{x^p(\alpha)s^p(\alpha)}{\mu(1-\alpha\theta)}\right) &= \min\left(v_+^2 + \frac{\alpha\theta}{1-\alpha\theta}\frac{w}{\mu} + \frac{\alpha^2\theta^2}{1-\alpha\theta}d_x^p d_s^p\right) \\ &\geq \min(v_+^2) + \frac{\alpha\theta}{1-\alpha\theta}\frac{\min(w)}{\mu} - \frac{\alpha^2\theta^2}{1-\alpha\theta}\left\|d_x^p d_s^p\right\|_\infty. \end{aligned}$$

Due to the fact that the WLCP is monotone, we have $(d_x^p)^T d_s^p \geq 0$. Therefore

$$\begin{aligned} \left\|d_x^p d_s^p\right\| &\leq \frac{1}{2\sqrt{2}}\left\|d_x^p + d_s^p\right\|^2 = \frac{1}{2\sqrt{2}}\left\|\frac{\frac{w}{\mu} - v_+^2}{v_+}\right\|^2 \\ &\leq \frac{1}{2\sqrt{2}\min(v_+^2)}\left\|\frac{w}{\mu} - v_+^2\right\|^2 \\ &\leq \frac{1}{2\sqrt{2}(\gamma - \delta^+)}\left\|\frac{w}{\mu} - v_+^2\right\|^2. \end{aligned} \tag{3.7}$$

Let us use the following notation:

$$\sigma^+ = \left\|\frac{w}{\mu} - v_+^2\right\|.$$

We have

$$\delta^+ = \left\|\frac{w(\mu)}{\mu} - v_+^2\right\| = \left\|\frac{w}{\mu} + \frac{1}{\mu^0}(c-w) - v_+^2\right\| \geq \left\|\frac{w}{\mu} - v_+^2\right\| - \frac{1}{\mu^0}\|c-w\|,$$

hence,

$$\sigma^{+} \leq \delta^{+} + \frac{1}{\mu^{0}}\|c-w\|. \tag{3.8}$$

Substitution of the bound (3.8) into (3.7) yields

$$\left\|d_x^p d_s^p\right\| \leq \frac{1}{2\sqrt{2}(\gamma-\delta^{+})}\left(\delta^{+} + \frac{1}{\mu^{0}}\|c-w\|\right)^{2}. \tag{3.9}$$

Therefore

$$\begin{aligned}
\min\left(\frac{x^p(\alpha)s^p(\alpha)}{\mu(1-\alpha\theta)}\right) &\geq \gamma-\delta^{+} + \frac{\alpha\theta}{1-\alpha\theta}\frac{\min(w)}{\mu} \\
&\qquad -\frac{\alpha^2\theta^2}{2\sqrt{2}(1-\alpha\theta)(\gamma-\delta^{+})}\left(\delta^{+} + \frac{1}{\mu^{0}}\|c-w\|\right)^{2} \\
&\geq \gamma-\delta^{+} \\
&\qquad -\frac{\alpha^2\theta^2}{2\sqrt{2}(1-\alpha\theta)(\gamma-\delta^{+})}\left(\delta^{+} + \frac{1}{\mu^{0}}\|c-w\|\right)^{2} \\
&\geq \gamma-\delta^{+} \\
&\qquad -\frac{\theta^2}{2\sqrt{2}(1-\theta)(\gamma-\delta^{+})}\left(\delta^{+} + \frac{1}{\mu^{0}}\|c-w\|\right)^{2} \\
&= h(\delta^{+},\theta,\gamma) > 0,
\end{aligned} \tag{3.10}$$

where the third inequality follows from the fact that

$$g(\alpha) = \frac{\alpha^2\theta^2}{1-\alpha\theta}$$

is strictly increasing for $\alpha \in [0,1]$. The inequality (3.10) implies that $x^p(\alpha)s^p(\alpha) > 0$ for $\alpha \in [0,1]$. Therefore, $x^p(\alpha)$ and $s^p(\alpha)$ do not change sign on $\alpha \in [0,1]$. Since $x^p(0) = x_+ > 0$ and $s^p(0) = s_+ > 0$, by continuity, we can conclude that $x^p(1) = x^p > 0$ and $s^p(1) = s^p > 0$. The proof is complete. ☐We define

$$v^p := \sqrt{\frac{x^p s^p}{\mu^p}}, \quad \text{where} \quad \mu^p = (1-\theta)\mu.$$

Substituting $\alpha = 1$ in (3.6) and (3.10) gives

$$\begin{aligned}
(v^p)^2 &= v_+^2 + \frac{\theta w}{(1-\theta)\mu} + \frac{\theta^2}{1-\theta}d_x^p d_s^p, \\
\min\left(v^p\right)^2 &\geq h(\delta^{+},\theta,\gamma) > 0.
\end{aligned} \tag{3.11}$$

In the next lemma, we examine the effect of a predictor step and the update of μ on the proximity measure.

Lemma 6. *Let (x_+, s_+, y_+) be an iterate obtained after a corrector step and $\mu^p = (1-\theta)\mu$ where $0 < \theta < 1$. Moreover, let (x^p, s^p, y^p) denote the iterate after a predictor step. Then, we have*

$$\delta^p := \delta(x^p, s^p; \mu^p) \leq \frac{2\theta}{1-\theta}\delta^+ + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| + \gamma - h(\delta^+, \theta, \gamma).$$

Proof By the definition of $w(\mu)$, we have

$$\begin{aligned} w(\mu^p) &= \Big(1 - \frac{\mu^p}{\mu^0}\Big)w + \frac{\mu^p}{\mu^0}c = \Big(1 - \frac{(1-\theta)\mu}{\mu^0}\Big)w + \frac{(1-\theta)\mu}{\mu^0}c \\ &= w(\mu) + \frac{\theta\mu}{\mu^0}(w-c). \end{aligned}$$

Therefore, by the definition of proximity measure and (3.11), we obtain

$$\begin{aligned} \delta^p &= \Big\|\frac{w(\mu^p)}{\mu^p} - (v^p)^2\Big\| \\ &= \Big\|\frac{w(\mu)}{(1-\theta)\mu} + \frac{\theta}{(1-\theta)\mu^0}(w-c) - v_+^2 - \frac{\theta w}{(1-\theta)\mu} - \frac{\theta^2}{1-\theta}d_x^p d_s^p\Big\| \\ &= \frac{1}{1-\theta}\Big\|\frac{w(\mu)}{\mu} + \frac{\theta}{\mu^0}(w-c) - v_+^2 + \theta v_+^2 - \frac{\theta w}{\mu} - \theta^2 d_x^p d_s^p\Big\| \\ &\leq \frac{1}{1-\theta}\Big(\Big\|\frac{w(\mu)}{\mu} - v_+^2\Big\| + \frac{\theta}{\mu^0}\|w-c\| + \theta\Big\|\frac{w}{\mu} - v_+^2\Big\| + \theta^2\|d_x^p d_s^p\|\Big) \\ &\leq \frac{1}{1-\theta}\Big(\delta^+ + \frac{\theta}{\mu^0}\|w-c\| + \theta\big(\delta^+ + \frac{1}{\mu^0}\|w-c\|\big) \\ &\qquad\qquad + \frac{\theta^2}{2\sqrt{2}(\gamma-\delta^+)}\big(\delta^+ + \frac{1}{\mu^0}\|w-c\|\big)^2\Big) \\ &= \frac{1+\theta}{1-\theta}\delta^+ + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| \\ &\qquad\qquad + \frac{\theta^2}{2\sqrt{2}(1-\theta)(\gamma-\delta^+)}\big(\delta^+ + \frac{1}{\mu^0}\|w-c\|\big)^2 \\ &= \Big(\frac{1+\theta}{1-\theta} - 1\Big)\delta^+ + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| + \gamma - h(\delta^+, \theta, \gamma) \\ &= \frac{2\theta}{1-\theta}\delta^+ + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| + \gamma - h(\delta^+, \theta, \gamma), \end{aligned}$$

where the second inequality is due to (3.8) and (3.9). The proof is complete. ☐In the sequel, we determine the proximity parameter τ and the update parameter $\bar{\theta}$ defined in (3.5). Let $(x, s, y) \in \mathcal{N}_2(\tau, \mu)$ be the iterate at the start of an iteration such that $\delta(x, s; \mu) \leq \tau \leq (1 - \frac{1}{2\sqrt{2}})\gamma$. After a corrector step, by Lemma 4, one has

$$\delta^+ := \delta(x_+, s_+; \mu) \leq \frac{\delta^2}{\gamma}.$$

It is clear that the right-hand side of the above inequality is increasing with respect to δ, so we have

$$\delta^+ \leq \frac{\tau^2}{\gamma}. \tag{3.12}$$

It can be verified that $h(\delta^+, \theta, \gamma)$ is decreasing with respect to δ^+. In this way, we get

$$h(\delta^+, \theta, \gamma) \geq h(\frac{\tau^2}{\gamma}, \theta, \gamma). \tag{3.13}$$

By invoking Lemma 6 and using (3.12) and (3.13) we deduce

$$\delta^p \leq \frac{2\theta}{1-\theta}\frac{\tau^2}{\gamma} + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| + \gamma - h\big(\frac{\tau^2}{\gamma}, \theta, \gamma\big).$$

To keep $\delta^p \leq \tau$, it suffices that

$$\frac{2\theta}{1-\theta}\frac{\tau^2}{\gamma} + \frac{2\theta}{(1-\theta)\mu^0}\|w-c\| + \gamma - h\big(\frac{\tau^2}{\gamma}, \theta, \gamma\big) \leq \tau. \tag{3.14}$$

At this stage, if we set $\tau = \frac{\gamma}{2}$ and

$$\theta = \frac{\min c}{12(\min c + \|w-c\|)}, \tag{3.15}$$

then $\theta \leq \frac{1}{12}$, which yields $\frac{1}{1-\theta} \leq \frac{12}{11}$, and by (3.12), $\delta^+ \leq \frac{\gamma^2}{4\gamma} = \frac{\gamma}{4} = \frac{\tau}{2}$. Moreover, we have

$$\theta\frac{\|w-c\|}{\mu^0} = \frac{(\min c)\|w-c\|}{12\mu^0(\min c + \|w-c\|)} = \frac{\gamma}{12} \times \frac{\|w-c\|}{\min c + \|w-c\|} \leq \frac{\gamma}{12} = \frac{\tau}{6},$$

and

$$\begin{aligned}
\gamma - h\big(\frac{\tau^2}{\gamma}, \theta, \gamma\big) &= \frac{\tau^2}{\gamma} + \frac{\theta^2}{2\sqrt{2}(1-\theta)(\gamma - \frac{\tau^2}{\gamma})}\big(\frac{\tau^2}{\gamma} + \frac{1}{\mu^0}\|w-c\|\big)^2 \\
&\leq \frac{\tau}{2} + \frac{1}{2\sqrt{2}(1-\theta)\frac{3\tau}{2}}\big(\theta\frac{\tau}{2} + \frac{\tau}{6}\big)^2 \\
&= \Big(1 + \frac{1}{6\sqrt{2}(1-\theta)}\big(\theta + \frac{1}{3}\big)^2\Big)\frac{\tau}{2} \\
&\leq \Big(1 + \frac{25}{792\sqrt{2}}\Big)\frac{\tau}{2}.
\end{aligned} \tag{3.16}$$

Therefore

$$\frac{2\theta}{1-\theta}\frac{\tau^2}{\gamma}+\frac{2\theta}{(1-\theta)\mu^0}\|w-c\|+\gamma-h(\frac{\tau^2}{\gamma},\theta,\gamma)\leq\Big(\frac{1}{11}+\frac{4}{11}+\frac{1}{2}+\frac{25}{1584\sqrt{2}}\Big)\tau\leq\tau,$$

which means that inequality (3.14) holds. On the other hand, from (3.13) and (3.16), we have

$$h(\delta^+,\theta,\gamma)\geq h(\frac{\tau^2}{\gamma},\theta,\gamma)\geq\gamma-\Big(1+\frac{25}{792\sqrt{2}}\Big)\frac{\tau}{2}=\Big(2-\frac{1}{2}-\frac{25}{1584\sqrt{2}}\Big)\tau>0.$$

Therefore, for $\bar{\theta}$ equal to θ as defined in (3.15), we have

$$(x^p,s^p,y^p)\in\mathcal{N}_2(\tau,\mu^p).$$

Now, we propose a predictor-corrector interior-point algorithm for solving WLCP. Given a weight vector $w>0$ and an initial point $(x^0,s^0,y^0)\in\mathcal{N}_2(\tau,\mu)$. Our algorithm performs a corrector step and a predictor step in one main iteration. In corrector step, we compute the search direction $(\Delta^c x,\Delta^c s,\Delta^c y)$ for WLCP by solving the system (3.1) and using (3.2). Then, we take a full-Newton step along this direction, which is the corrector iterate (x_+,s_+,y_+). Then we calculate the predictor direction $(\Delta^p x,\Delta^p s,\Delta^p y)$ by solving (3.3) and using (3.4). So, we consider the predictor iterate (x^p,s^p,y^p). We repeat the process until an iterate satisfying the stopping criterion $\|x^k s^k-w\|\leq\varepsilon$. The predictor-corrector algorithm is presented below.

Algorithm 1: Predictor − corrector algorithm for WLCP

Input: Required accuracy parameter $\varepsilon>0$; threshold parameter $0<\tau<1$ (default $\tau=\frac{\gamma}{2}$);
barrier update parameter $0<\theta<1$ (default (3.15)); an initial point
$(x^0,s^0,y^0)\in\mathcal{N}_2(\tau,\mu)$; Consider the notation from (2.1).
begin
$k:=0$;
while $\|x^k s^k-w\|>\varepsilon$ **do**
begin
(corrector step)
compute $(\Delta^c x^k,\Delta^c s^k,\Delta^c y^k)$ from system (3.1) using (3.2);
let $(x_+^k,s_+^k,y_+^k):=(x^k,s^k,y^k)+(\Delta^c x^k,\Delta^c s^k,\Delta^c y^k)$;
(predictor step)
compute $(\Delta^p x^k,\Delta^p s^k,\Delta^p y^k)$ from system (3.3) using (3.4);
let $((x^p)^k,(s^p)^k,(y^p)^k):=(x_+^k,s_+^k,y_+^k)+\theta(\Delta^p x^k,\Delta^p s^k,\Delta^p y^k)$;
(update of the iterates)
$(x^{k+1},s^{k+1},y^{k+1}):=((x^p)^k,(s^p)^k,(y^p)^k)$;
$\mu^{k+1}:=(1-\theta)\mu^k$;
$k:=k+1$;
end
end.

4. Iteration Bound

Lemma 7. *Let (x, s, y) be an iterate obtained by Algorithm 1 such that $\delta := \delta(x, s; \mu) \leq (1 - \frac{1}{2\sqrt{2}})\gamma$. Moreover, let (x^p, s^p, y^p) be an iterate obtained after a predictor step and $\mu^p = (1-\theta)\mu$ where $0 < \theta < 1$. Then*

$$\|x^p s^p - w\| \leq \Big(1 - \frac{\theta}{2}\Big)\mu\beta < \beta\frac{\mu^p}{1-\theta}.$$

where

$$\beta = \max\left\{\frac{\delta^2}{\gamma} + \frac{\|c-w\|}{\mu^0}, \frac{\gamma}{\sqrt{2}(\gamma^2 - \delta^2)}\Big(\frac{\delta^2}{\gamma} + \frac{1}{\mu^0}\|c-w\|\Big)^2\right\}.$$

Proof Using the definition of v^p and (3.11), we have

$$\begin{aligned} x^p s^p = \mu^p (v^p)^2 &= (1-\theta)\mu\big(v_+^2 + \frac{\theta w}{(1-\theta)\mu} + \frac{\theta^2}{1-\theta} d_x^p d_s^p\big) \\ &= (1-\theta)\mu v_+^2 + \theta w + \mu\theta^2 d_x^p d_s^p. \end{aligned}$$

Therefore, we have

$$\begin{aligned} x^p s^p - w &= (1-\theta)\big(\mu v_+^2 - w\big) + \mu\theta^2 d_x^p d_s^p \\ &= (1-\theta)\Big(\mu v_+^2 - w(\mu) + \frac{\mu}{\mu^0}(c-w)\Big) + \mu\theta^2 d_x^p d_s^p \\ &= (1-\theta)\mu\Big(v_+^2 - \frac{w(\mu)}{\mu} + \frac{1}{\mu^0}(c-w)\Big) + \mu\theta^2 d_x^p d_s^p, \end{aligned} \tag{4.1}$$

where the second equality is due to (2.1). By taking the norm of both sides of (4.1), we get

$$\begin{aligned} \|x^p s^p - w\| &= \Big\|(1-\theta)\mu\Big(v_+^2 - \frac{w(\mu)}{\mu} + \frac{1}{\mu^0}(c-w)\Big) + \mu\theta^2 d_x^p d_s^p\Big\| \\ &\leq (1-\theta)\mu\Big(\Big\|v_+^2 - \frac{w(\mu)}{\mu}\Big\| + \frac{1}{\mu^0}\|c-w\|\Big) + \mu\theta^2\big\|d_x^p d_s^p\big\| \\ &\leq (1-\theta)\mu\Big(\delta^+ + \frac{1}{\mu^0}\|c-w\|\Big) + \frac{\mu\theta^2}{2\sqrt{2}(\gamma - \delta^+)}\Big(\delta^+ + \frac{1}{\mu^0}\|c-w\|\Big)^2 \\ &\leq (1-\theta)\mu\Big(\frac{\delta^2}{\gamma} + \frac{1}{\mu^0}\|c-w\|\Big) + \frac{\mu\theta^2\gamma}{2\sqrt{2}(\gamma^2 - \delta^2)}\Big(\frac{\delta^2}{\gamma} + \frac{1}{\mu^0}\|c-w\|\Big)^2 \\ &\leq \Big(1 - \theta + \frac{\theta^2}{2}\Big)\mu\beta < \Big(1 - \frac{\theta}{2}\Big)\mu\beta, \end{aligned}$$

where the second inequality is due to the definition of δ^+ and (3.9) and the third inequality follows from Lemma 4. Using $\mu^p = (1-\theta)\mu$ and the above inequality, we obtain

$$\big\|x^p s^p - w\| \leq \Big(1 - \frac{\theta}{2}\Big)\beta\frac{\mu^p}{1-\theta} < \beta\frac{\mu^p}{1-\theta},$$

which completes the proof. □

Theorem 1. *Let $(x^0, s^0, y^0) \in \mathcal{N}_2(\frac{\gamma}{2}, \mu)$ and θ is defined as in (3.15). Moreover, let (x^k, s^k, y^k) be the iterate obtained after k iterations. Then,*

$$\|x^k s^k - w\| \leq \varepsilon$$

for

$$k \geq 1 + \left\lceil \frac{1}{\theta} \log \frac{\beta_1 \mu^0}{\varepsilon} \right\rceil,$$

where $\beta_1 := \max\left\{\frac{\gamma}{4} + \frac{\|c-w\|}{\mu^0}, \frac{2\sqrt{2}}{3\gamma}\left(\frac{\gamma}{4} + \frac{1}{\mu^0}\|c-w\|\right)^2\right\}$.

Proof From Lemma 7 it follows that

$$\|x^k s^k - w\| < \beta \frac{\mu^k}{1-\theta} = \beta(1-\theta)^{k-1}\mu^0.$$

Due to $\delta \leq \tau = \frac{\gamma}{2}$, we have

$$\beta \leq \beta_1.$$

Hence, the convergence criterion $\|x^k s^k - w\| \leq \varepsilon$ is satisfied if we have

$$\beta_1(1-\theta)^{k-1}\mu^0 \leq \varepsilon. \tag{4.2}$$

By taking logarithms of both sides in (4.2), we obtain

$$(k-1)\log(1-\theta) + \log(\beta_1\mu^0) \leq \log\varepsilon.$$

Using the inequality $\log(1+x) \leq x, x > -1$, we conclude that the above inequality holds for all k that satisfy

$$k \geq 1 + \left\lceil \frac{1}{\theta} \log \frac{\beta_1 \mu^0}{\varepsilon} \right\rceil,$$

which proves the lemma. □

Theorem 2. *Let $\tau = \frac{\gamma}{2}$ and let θ be defined as in (3.15). Then, Algorithm 1 is well-defined and requires at most*

$$O\Big(\frac{\min c + \|w - c\|}{\min c} \log \frac{\beta_1 \mu^0}{\varepsilon}\Big)$$

iterations in order to get $\|x^k s^k - w\| \leq \varepsilon$.

Corollary 1. *If WLCP (1.1) is monotone, then Algorithm 1 finds an ε-approximate solution for this problem in at most*

$$O\Big(\frac{\gamma(x^0)^T s^0/n + \|w - x^0 s^0\|}{\min x^0 s^0} \log \frac{\beta_1 (x^0)^T s^0/n}{\varepsilon}\Big)$$

iterations.

Proof From Theorem 2 and (2.1), the result is easily obtained. □

5. Numerical Results

To illustrate the validity and behavior of the Algorithm 1 (Algor. 1), in this section the Algor. 1 is compared with the proposed algorithm in [12] (Algor. 2) on the small set of randomly generated problems. It is shown that it behaves well on this set of problems. First, matrices R, Q and matrix $A \in \mathbf{R}^{n\times n}$ are generated with entries randomly selected from the interval $[-9, 9]$. Then the matrix P is considered as $P = -Q(A^T A)$. The entries of the weight vector w are randomly selected from the interval $[0.1, 0.9]$. For these WLCPs, we also selected an initial starting point (x^0, s^0, y^0) such that $(x^0, s^0, y^0) \in \mathcal{N}_2(\frac{\gamma}{2}, \mu)$, where $\gamma = \frac{\min x^0 s^0}{\mu^0}$ with $\mu^0 = \frac{(x^0)^T s^0}{n}$. We set $\varepsilon = 10^{-8}$. For each n, m, p, the entry in the column "Iter" is the average number of iterations of 10 randomly generated WLCPs with the same dimensions. The results are summarized in Table 1, which shows that Algorithm1 performs better that Algorithm2 in terms of the number of iterations.

Table 1. Numerical results

n	m	p	Iter(Algor. 1)	Iter(Algor. 2)	n	m	p	Iter(Algor. 1)	Iter(Algor. 2)
10	7	5	75.2	126.7	30	20	15	88.9	132.8
50	40	25	87.2	135.1	100	80	60	115.1	137.8
300	200	200	104.3	141.9	500	400	200	95.5	143.2
350	250	300	133.5	161.3	450	350	300	147.2	179.4
600	500	300	92.7	123.5	700	600	500	100.8	132.7

6. Concluding Remarks

In this paper, we have presented a predictor-corrector IPM for solving monotone WLCP that operates in a τ-neighborhood of the central path. We analyzed this small-update algorithm and proved that the proposed algorithm is feasible. The corrector stage is based on full Newton's steps while the predictor step uses damped step. With appropriate choices of the barrier update parameter θ and the threshold parameter τ, iterates are shown to stay in the $\mathcal{N}_2(\tau, \mu)$ neighborhood of the central path. Furthermore, the derived iteration bound for the Algorithm 1 matches the currently best known iteration bound for monotone WLCPs. Numerical results indicate that the Algorithm 1 performs well on a small set of test problems.
One direction for future research could be to consider the generalization of this IPM to weighted linear complementarity problem over symmetric cones.

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

References

[1] M. Achache, *A new primal-dual path-following method for convex quadratic programming*, Comput. Appl. Math. **25** (2006), no. 1, 97–110. https://doi.org/10.1590/S0101-82052006000100005.

[2] K.M. Anstreicher, *Interior-point algorithms for a generalization of linear programming and weighted centering*, Optim. Methods Soft. **27** (2012), no. 4-5, 605–612. https://doi.org/10.1080/10556788.2011.644791.

[3] S. Asadi, Z. Darvay, G. Lesaja, N. Mahdavi-Amiri, and F.A. Potra, *A full-Newton step interior-point method for monotone weighted linear complementarity problems*, J. Optim. Theory Appl. **186**, no. 3, 864–878. https://doi.org/10.1007/s10957-020-01728-4.

[4] R.W. Cottle, J.S. Pang, and R.E. Stone, *The Linear Complementarity Problem*, Academic Press, San Diego, CA, 1992.

[5] Z. Darvay, *New interior-point algorithm in linear programming*, Adv. Model. Optim. **5** (2003), 51–92.

[6] Z. Darvay, T. Illés, B. Kheirfam, and P.R. Rigó, *A corrector-predictor interior-point method with new search direction for linear optimization*, Cent. Euro. J. Oper. Res. **28** (2020), no. 3, 1123–1140. https://doi.org/10.1007/s10100-019-00622-3.

[7] Z. Darvay, T. Illés, J. Povh, and P.R. Rigó, *Feasible corrector-predictor interior-point algorithm for $P_*(\kappa)$-linear complementarity problems based on a new search direction*, SIAM J. Optim. **30** (2020), no. 3, 2628–2658. https://doi.org/10.1137/19M1248972.

[8] Z. Darvay, T. Illés, and P.R. Rigó, *Predictor-corrector interior-point algorithm for $p_*(\kappa)$-linear complementarity problems based on a new type of algebraic equivalent transformation technique*, Euro. J. Oper. Res. **298** (2022), no. 7, 25–35. https://doi.org/10.1016/j.ejor.2021.08.039.

[9] B. Kheirfam, *A corrector-predictor path-following algorithm for semidefinite optimization*, Asian-Eur. J. Math. **7** (2014), no. 2, 1450028. https://doi.org/10.1142/S1793557114500284.

[10] ———, *A predictor-corrector interior-point algorithm for $P_*(\kappa)$ horizontal linear complementarity problem*, Numer. Algorithms **66** (2014), no. 2, 349–361. https://doi.org/10.1007/s11075-013-9738-3.

[11] ———, *A corrector-predictor path-following method for convex quadratic symmetric cone optimization*, J. Optim. Theory Appl. **164** (2015), no. 1, 246–260. https://doi.org/10.1007/s10957-014-0554-2.

[12] ———, *Complexity analysis of a full-Newton step interior-point method for monotone weighted linear complementarity problems*, J. Optim. Theory Appl. **202** (2024), no. 1, 133–145. https://doi.org/10.1007/s10957-022-02139-3.

[13] K.A. McShane, *Superlinearly convergent $o(\sqrt{n}l)$-iteration interior-point algorithms for linear programming and the monotone linear complementarity problem*, SIAM J. Optim. **4** (1994), no. 2, 247–261.
https://doi.org/10.1137/0804014.

[14] S. Mizuno, M.J. Todd, and Y. Ye, *On adaptive-step primal-dual interior-point algorithms for linear programming*, Math. Oper. Res. **18**, no. 4, 964–981.
https://doi.org/10.1287/moor.18.4.964.

[15] F.A. Potra, *Corrector-predictor methods for monotone linear complementarity problems in a wide neighborhood of the central path*, Math. Program. **111** (2008), 243–272.
https://doi.org/10.1007/s10107-006-0068-2.

[16] ______, *Weighted complementarity problems- a new paradigm for computing equilibria*, SIAM J. Optim. **22** (2012), no. 4, 1634–1654.
https://doi.org/10.1137/110837310.

[17] ______, *Sufficient weighted complementarity problems*, Comput. Optim. Appl. **64** (2016), no. 2, 467–488.
https://doi.org/10.1007/s10589-015-9811-z.

[18] F.A. Potra and X. Liu, *Predictor-corrector methods for sufficient linear complementarity problems in a wide neighborhood of the central path*, Optim. Methods Softw. **20** (2005), no. 1, 145–168.
https://doi.org/10.1080/10556780512331318038.

[19] C. Roos, T. Terlaky, and J.-Ph. Vial, *Theory and Algorithms for Linear Optimization. An Interior-Point Approach*, John Wiley & Sons, Chichester, UK, 1997.

[20] J.Y. Tang and J.C. Zhou, *A modified damped Gauss-Newton method for non-monotone weighted linear complementarity problems*, Optim. Methods Softw. **37** (2022), no. 3, 1145–1164.
https://doi.org/10.1080/10556788.2021.1903007.

[21] G.Q. Wang and Y.Q. Bai, *A primal-dual interior-point algorithm for second-order cone optimization with full Nesterov-Todd step*, Appl. Math. Comput. **215** (2009), no. 3, 1047–1061.
https://doi.org/10.1016/j.amc.2009.06.034.

[22] ______, *A new full Nesterov-Todd step primal–dual path-following interior-point algorithm for symmetric optimization*, J. Optim. Theory Appl. **154** (2012), 966–985.
https://doi.org/10.1007/s10957-012-0013-x.

[23] G.Q. Wang, C.J. Yu, and K.L. Teo, *A full-Newton step feasible interior-point algorithm for $P_*(\kappa)$-linear complementarity problems*, J. Glob. Optim. **59** (2014), no. 1, 81–99.
https://doi.org/10.1007/s10898-013-0090-x.

[24] Y. Ye, *A fully polynomial-time approximation algorithm for computing a stationary point of the general linear complementarity problem*, Math. Oper. Res. **18** (1993), no. 2, 334–345.