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Research Article

Eccentric adjacency index of graph operations and its applications

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Abstract: The study of topological descriptors is very beneficial in determining the underlying topologies of graphs and networks. An extensive collection of graph-associated numerical descriptors has been used to examine the whole structure of networks. In this analysis, eccentricity-based topological indices have secured a significant place in theoretical chemistry and nanotechnology. Also, graph products conveniently play an essential role in many combinatorial applications, graph decompositions, pure mathematics, and applied mathematics. In this article, we derive the precise results for the eccentric adjacency index of some graph products such as composition, Indu-Bala, Cartesian, disjunction, and symmetric difference products. Furthermore, we implement these outcomes to deduce the eccentric adjacency index for certain significant classes of chemical structures in the factors of graph products. The chemical significance of the index is also investigated.

Keywords: topological indices, eccentric adjacency index, graph operations, QSPR analysis.

AMS Subject classification: 05C50, 05C92

1. Introduction

Throughout the paper, all graphs are connected and simple. For a given graph $\mathbb{H}$, the sets $\mathcal{E}(\mathbb{H})$ and $\mathcal{V}(\mathbb{H})$ denotes the edge and the vertex sets, respectively. For $\mathbb{H}$, the

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size and order are denoted by t and s, respectively. The degree of a vertex $w \in \mathcal{V}(\mathbb{H})$ is the number of adjacent edges to w in $\mathbb{H}$ and written as $\deg_{\mathbb{H}}(w)$. For $w \in \mathcal{V}(\mathbb{H})$, $\mathbb{N}_{\mathcal{H}}(w)$ is the set of all adjacent vertices of w in $\mathbb{H}$ and $\omega_{\mathbb{H}}(w) = \sum_{w' \in \mathbb{N}(w)} \deg_{\mathbb{H}}(w')$. For vertices $w, w' \in \mathcal{V}(\mathbb{H})$, the distance $d_{\mathbb{H}}(w, w')$ is the length of smallest path from w to w' in $\mathbb{H}$ and eccentricity $ecc_{\mathbb{H}}(w)$ is defined as $ecc_{\mathbb{H}}(w) = \max_{w' \in \mathcal{V}(\mathbb{H})} d_{\mathbb{H}}(w, w')$. The radius $r_{\mathbb{H}}$ and the diameter $d_{\mathbb{H}}$ of $\mathbb{H}$ are described by $r_{\mathbb{H}} = \min_{w \in \mathcal{V}(\mathbb{H})} ecc_{\mathbb{H}}(w)$ and $d_{\mathbb{H}} = \max_{w \in \mathcal{V}(\mathbb{H})} ecc_{\mathbb{H}}(w)$, respectively. The center of $\mathbb{H}$ is the subgraph of it, induced by the set of vertices with minimum eccentricity and is presented by $C(\mathbb{H})$. A graph $\mathbb{H}$ is said to be a self-centered graph if $\mathbb{H} \cong C(\mathbb{H})$. The vertices with degree $s-1$ and eccentricity 1 in $\mathbb{H}$ are known as the well-connected vertices in $\mathbb{H}$ and the set of all such vertices is denoted as $\Gamma(\mathbb{H})$. The graph K_1 with unique vertex is considered to be a well-connected graph. The notions C_s, K_s and P_s are used for the cycle, complete graph and path, respectively.

Graph theory have a number of applications in chemistry such as proteomics, Quantitative structure-property and activity correlations, graph polynomials for analysis of structures, isomer enumeration, quantum chemistry, spectroscopy, numarical and other procedures for prediction of toxicity of chemical compounds [4, 8, 14–17, 26–29, 31]. The QSAR/QSPR studies develop correlation between the properties of chemical compounds and molecular connectivity, therefore graph-theoretical properties develop the principles for the drug discovery and the computer-aided predictive toxicology. As a consequence, successful uses of QSPR/QSAR studies have stimulated the emergence of various topological invariants of chemical graphs [9, 26–31, 35–37]. The intermolecular interactions depend upon the degree criterions, distance and moreover, a number of physico-chemical characteristics of chemical compounds have been proven to interrelate with topological indices as a decent initial points. A topological index is a numeric quantity that characterizes the properties like guest-host interactions, receptor binding propensity, protein-drug interactions, toxicity, dermal penetrations, drug metabolomics, etc., of a molecular graph. Therefore, topological indices are much appealing tool of statistical approximation for securing structure-activity relations. There are many classes of topological indices; such as distance, degree and eccentricity based indices of graphs.

The Zagreb indices are the famous molecular descriptors, and these indices have recognizable applications in chemistry. In 1972, Gutman and Trinajstić [25] introduced the first Zagreb index which is based upon the degree of vertices in $\mathbb{H}$. The first and second Zagreb indices of $\mathbb{H}$ can be described as:

$$\mathcal{M}_1(\mathbb{H}) = \sum_{w \in \mathcal{V}(\mathbb{H})} \deg^2_{\mathbb{H}}(w), \quad \mathcal{M}_2(\mathbb{H}) = \sum_{ww' \in \mathcal{E}(\mathbb{H})} \deg_{\mathbb{H}}(w) \deg_{\mathbb{H}}(w').$$

In the present-day literature, many eccentricity based indices have been presented; one of the most known index is the eccentric connectivity index. The eccentric connectivity index contribute in the foreseeability of pharmaceutical characteristics and furnish leads for the establishment of safe and applicable anti-HIV compounds [19]. Sharma et al. [40] defined the eccentric connectivity index as follows:

$$\xi^c(\mathbb{H}) = \sum_{w\in\mathcal{V}(\mathbb{H})} \deg_{\mathbb{H}}(w) ecc_{\mathbb{H}}(w).$$

The connective eccentricity and the total eccentricity indices of $\mathbb{H}$ are the modification of eccentric connectivity index (see [23]). These are described as follows:

$$\xi^{ce}(\mathbb{H}) = \sum_{w\in\mathcal{V}(\mathbb{H})} \frac{\deg_{\mathbb{H}}(w)}{ecc_{\mathbb{H}}(w)}, \quad \tau(\mathbb{H}) = \sum_{w\in\mathcal{V}(\mathbb{H})} ecc_{\mathbb{H}}(w).$$

Also, the modification of connective eccentricity and eccentric connectivity indices is known as the eccentric adjacency index [20, 24] and can be described as follows:

$$\xi^{ad}(\mathbb{H}) = \sum_{w\in\mathcal{V}(\mathbb{H})} \frac{\omega_{\mathbb{H}}(w)}{ecc_{\mathbb{H}}(w)}. \tag{1.1}$$

Gupta et al. [24] examined the interconnection of anti-HIV activity of HEPT derivatives with the first order molecular eccentric adjacency and connectivity indices. The exactness of prediction of $\xi^{ad}(\mathbb{H})$ is more than ninety percent, thus it designs an extensive potential for the studies of QSAR/QSPR. The implementation of this index makes it noteworthy to investigate their mathematical characteristics and it is much appealing for the mathematicians to research the insights of this descriptor.
Now we represent a new index which is modification of total eccentricity index known as the inverse total eccentricity index [34] of $\mathbb{H}$. It is described as:

$$\tau^{-1}(\mathbb{H}) = \sum_{w\in\mathcal{V}(\mathbb{H})} \frac{1}{ecc_{\mathbb{H}}(w)}.$$

By different graph operations, one can design a new graph from given graphs, and also some interesting chemical graphs can be obtained as an outcome of these graph operations of some known graphs. The correlations attained for various characteristics of graph operations in the form of characteristics of their respective components, it is a worthwhile tool in the discussion of characteristics of some nanostructures and molecular graphs. There are vast studies regarding the characteristics of graph operations, for instance; see [1–3, 5–7, 10, 12, 13, 18, 21, 22, 32–34, 38, 41].
In this paper, we are interested to find eccentric adjacency index of some graph operations. Then, we perform our results to compute the eccentric adjacency index of certain significant categories of graphs in the type of elements of graph operations. The chemical significance of the index is also explored.
Before the discussion of our main results, let us give the first Zagreb index, eccentric adjacency index, connective eccentricity index and inverse total eccentricity index of P_s, C_s and K_s.

$$\mathcal{M}_1(P_s)=4s-6,\quad \mathcal{M}_1(C_s)=4s,\quad \mathcal{M}_1(K_s)=s(s-1)^2,$$
$$\xi^{ad}(C_s)=\frac{4s}{\left\lfloor\frac{s}{2}\right\rfloor},\quad \xi^{ce}(C_s)=\frac{2s}{\left\lfloor\frac{s}{2}\right\rfloor},\quad \tau^{-1}(C_s)=\frac{s}{\left\lfloor\frac{s}{2}\right\rfloor},$$
$$\xi^{ad}(K_s)=s(s-1)^2,\quad \xi^{ce}(K_s)=s,\quad \tau^{-1}(K_s)=s(s-1).$$

Proposition 1. [11, 20, 33, 34] *For the graphs P_s, C_s and K_s, we have*

$$\xi^{ad}(P_s)=\begin{cases}2 & \text{if } s=2,\\ 4 & \text{if } s=4,\\ 2(\frac{2}{s-1}+\frac{3}{s-2}+\frac{4}{s-3}+\frac{4}{s-4}+\cdots+\frac{4}{\lfloor s/2\rfloor+1})+\frac{4}{\lfloor s/2\rfloor} & \text{if } s>3 \text{ is odd},\\ 2(\frac{2}{s-1}+\frac{3}{s-2}+\frac{4}{s-3}+\frac{4}{s-4}+\cdots+\frac{4}{s/2}) & \text{if } s>4 \text{ is even}.\end{cases}$$

$$\xi^{ce}(P_s)=\begin{cases}\dfrac{2}{s-1}+4(\frac{1}{s-2}+\frac{1}{s-3}+\cdots+\frac{1}{\lfloor s/2\rfloor+1})+\frac{2}{\lfloor s/2\rfloor} & \text{if } s \text{ is odd},\\ \dfrac{2}{s-1}+4(\frac{1}{s-1}+\frac{1}{s-2}+\cdots+\frac{4}{s/2}) & \text{if } s \text{ is even}.\end{cases}$$

$$\tau^{-1}(P_s)=\begin{cases}2(\frac{1}{s-1}+\frac{1}{s-2}+\frac{1}{s-3}+\cdots+\frac{1}{\lfloor s/2\rfloor+1})+\frac{1}{\lfloor s/2\rfloor} & \text{if } s \text{ is odd},\\ 2(\frac{1}{s-1}+\frac{1}{s-2}+\frac{1}{s-3}+\cdots+\frac{1}{s/2}) & \text{if } s \text{ is even}.\end{cases}$$

2. Composition of graphs

The composition of $\mathbb{H}_1$ and $\mathbb{H}_2$ graphs is represented by $\mathbb{H}_1[\mathbb{H}_2]$. Its vertex set is $\mathcal{V}(\mathbb{H}_1)\times\mathcal{V}(\mathbb{H}_2)$ and $(w_1,w_2)(w_1',w_2')\in\mathcal{E}(\mathbb{H}_1[\mathbb{H}_2])$ if $w_1w_1'\in\mathcal{E}(\mathbb{H}_1)$ or $w_1=w_1'$ and $w_2w_2'\in\mathcal{E}(\mathbb{H}_1)$. Next lemma gives some properties, which will be worthwhile in our leading result.

Lemma 1. *Let $\mathbb{H}_1\ncong K_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

(a) $\omega_{\mathbb{H}_1[\mathbb{H}_2]}((w_1,w_2))=s_2^2\omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2t_2\deg_{\mathbb{H}_1}(w_1)+s_2\deg_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2)$.

(b) $$ecc_{\mathbb{H}_1[\mathbb{H}_2]}((w_1,w_2))=\left\{\begin{array}{ll}ecc_{\mathbb{H}_1}(w_1), & \text{if } w_1\notin\Gamma(\mathbb{H}_1),\\ 1, & \text{if } w_1\in\Gamma(\mathbb{H}_1), w_2\in\Gamma(\mathbb{H}_2),\\ 2, & \text{if } w_1\in\Gamma(\mathbb{H}_1), w_2\notin\Gamma(\mathbb{H}_2).\end{array}\right\}$$

In the upcoming result, we derive the closed formula of the composition of $\mathbb{H}_1$ and $\mathbb{H}_2$.

Theorem 1. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

$$\xi^{ad}(\mathbb{H}_1[\mathbb{H}_2])=s_2^3\xi^{ad}(\mathbb{H}_1)+\frac{s_2^2}{2}(|\Gamma(\mathbb{H}_2)|-s_2)\sum_{w_1\in\Gamma(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)+\left(\tau^{-1}(\mathbb{H}_1)-\frac{1}{2}|\Gamma(\mathbb{H}_1)|\right)\mathcal{M}_1(\mathbb{H}_2)$$
$$+4s_2t_2\xi^{ce}(\mathbb{H}_1)+\frac{|\Gamma(\mathbb{H}_1)|}{2}\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)-2s_2t_2(s_1-1)|\Gamma(\mathbb{H}_1)|+\frac{1}{2}(s_1-1)$$
$$|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|\left(4t_2+s_2(s_2-1)\right).$$

Proof. By using Lemma 1 in (1.1), we obtain

$$
\begin{aligned}
&\xi^{ad}(\mathbb{H}_1[\mathbb{H}_2])\\
&= \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\backslash\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)} \frac{\begin{array}{r} n_2^2\omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2t_2\deg_{\mathbb{H}_1}(w_1)\\ +s_2\deg_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2)\end{array}}{ecc_{\mathbb{H}_1}(w_1)}\\
&+\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\Gamma(\mathbb{H}_2)} \frac{s_2^2\omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2t_2(s_1-1)+s_2(s_1-1)(s_2-1)}{1}\\
&+\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\backslash\Gamma(\mathbb{H}_2)} \frac{s_2^2\omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2t_2(s_1-1)+s_2(s_1-1)\deg_{\mathbb{H}_2}(w_2)}{2}\\
&= s_2^2\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\backslash\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\frac{\omega_{\mathbb{H}_1}(w_1)}{ecc_{\mathbb{H}_1}(w_1)}+\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\backslash\Gamma(\mathbb{H}_1)}\frac{1}{ecc_{\mathbb{H}_1}(w_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)\\
&+2t_2\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\backslash\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\frac{\deg_{\mathbb{H}_1}(w_1)}{ecc_{\mathbb{H}_1}(w_1)}+s_2\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\backslash\Gamma(\mathbb{H}_1)}\frac{\deg_{\mathbb{H}_1}(w_1)}{ecc_{\mathbb{H}_1}(w_1)}\\
&\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\deg_{\mathbb{H}_2}(w_2)+s_2^2\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_1}(w_1)+\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)\\
&+\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\Gamma(\mathbb{H}_2)}(2t_2(s_1-1)+s_2(s_1-1)(s_2-1))\\
&+\frac{s_2^2}{2}\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\backslash\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_1}(w_1)+\frac{1}{2}\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\backslash\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)\\
&+\frac{1}{2}\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\backslash\Gamma(\mathbb{H}_2)}2t_2(s_1-1)+\frac{s_2(s_1-1)}{2}\sum_{w_1\in\Gamma(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\backslash\Gamma(\mathbb{H}_2)}\deg_{\mathbb{H}_2}(w_2)\\
&= s_2^3\left(\xi^{ad}(\mathbb{H}_1)-\sum_{w_1\in\Gamma(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)\right)+\left(\tau^{-1}(\mathbb{H}_1)-|\Gamma(\mathbb{H}_1)|\right)\mathcal{M}_1(\mathbb{H}_2)+2t_2s_2(\xi^{ce}(\mathbb{H}_1)\\
&-(s_1-1)|\Gamma(\mathbb{H}_1)|)+2t_2s_2(\xi^{ce}(\mathbb{H}_1)-(s_1-1)|\Gamma(\mathbb{H}_1)|)+s_2^2|\Gamma(\mathbb{H}_2)|\sum_{w_1\in\Gamma(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)\\
&+|\Gamma(\mathbb{H}_1)|\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)+|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|(2t_2(s_1-1)+s_2(s_1-1)(s_2-1))\\
&+\frac{s_2^2}{2}(s_2-|\Gamma(\mathbb{H}_2)|)\sum_{w_1\in\Gamma(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)+\frac{|\Gamma(\mathbb{H}_1)|}{2}\left(\mathcal{M}_1(\mathbb{H}_2)-\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)\right)\\
&+t_2(s_1-1)|\Gamma(\mathbb{H}_1)|\left(s_2-|\Gamma(\mathbb{H}_2)|\right)+\frac{1}{2}s_2(s_1-1)|\Gamma(\mathbb{H}_1)|\left(2t_2-(s_2-1)|\Gamma(\mathbb{H}_2)|\right)\\
&= s_2^3\xi^{ad}(\mathbb{H}_1)+\frac{s_2^2}{2}(|\Gamma(\mathbb{H}_2)|-s_2)\sum_{w_1\in\Gamma(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)+\left(\tau^{-1}(\mathbb{H}_1)-\frac{1}{2}|\Gamma(\mathbb{H}_1)|\right)\mathcal{M}_1(\mathbb{H}_2)\\
&+4s_2t_2\xi^{ce}(\mathbb{H}_1)+\frac{|\Gamma(\mathbb{H}_1)|}{2}\sum_{w_2\in\Gamma(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)-2s_2t_2(s_1-1)|\Gamma(\mathbb{H}_1)|\\
&+2t_2(s_1-1)|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|+\frac{1}{2}s_2(s_1-1)(s_2-1)|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|.
\end{aligned}
$$

This finishes the proof. □

Example 1. The Catlin graph is the composition of C_5 and K_3, with $|\Gamma(C_5)| = 0$, $\sum_{w_1\in\Gamma(C_5)} \omega_{C_5}(w_1) = 0$, $|\Gamma(K_3)| = 3$ and $\sum_{w_2\in\Gamma(K_3)} \omega_{K_3}(w_2) = 12$, has the eccentric adjacency index 480, by the use of Theorem 1.

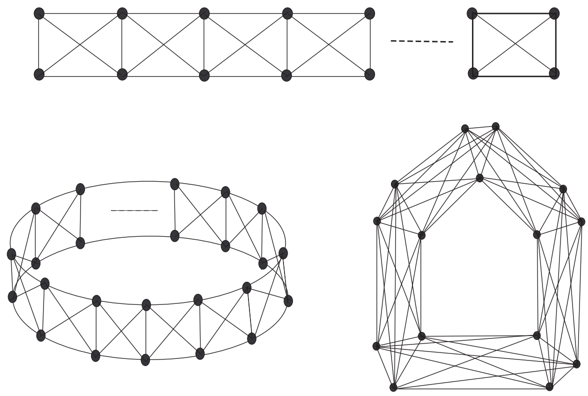

Figure 1. **The graphs $P_s[P_2]$, $C_s[P_2]$ and $C_5[K_3]$.**

Example 2. The fence graph is the composition of P_s and P_2 with $|\Gamma(P_s)| = 0$ for $s \geq 3$, $\sum_{w_1\in\Gamma(P_s)} \omega_{P_s}(w_1) = 0$ for $s \geq 3$, $|\Gamma(P_2)| = 2$, $\sum_{w_2\in\Gamma(P_2)} \omega_{P_2}(w_2) = 2$. From Theorem 1 and Proposition 1, we have

$$\xi^{ad}(P_s[P_2]) = \begin{cases} 36, & \text{if } s = 2, \\ 60, & \text{if } s = 3, \\ \dfrac{52s}{s-1} + \dfrac{84}{s-2} + 100\left(\dfrac{1}{s-3} + \dfrac{1}{s-4} + \cdots + \dfrac{1}{\left\lfloor \frac{s}{2} \right\rfloor}\right), & \text{if } s > 2 \text{ is even}, \\ \dfrac{52s}{s-1} + \dfrac{84}{s-2} + 100\left(\dfrac{1}{s-3} + \dfrac{1}{s-4} + \cdots + \dfrac{1}{\left\lfloor \frac{s}{2} \right\rfloor + 1}\right) + & \\ \dfrac{50}{\left\lfloor \frac{s}{2} \right\rfloor + 1}, & \text{if } s > 3 \text{ is odd}. \end{cases}$$

Example 3. The closed fence graph is the composition of C_s and P_2, with $|\Gamma(C_s)| = 0$, $\sum_{w_1\in\Gamma(C_s)} \omega_{C_s}(w_1) = 0$, $|\Gamma(P_2)| = 2$, $\sum_{w_2\in\Gamma(P_2)} \omega_{P_2}(w_2) = 2$, for $s \geq 4$. Based on Theorem 1 and Proposition 1, we get $\xi^{ad}(C_s[P_2]) = \dfrac{50s}{\left\lfloor \frac{s}{2} \right\rfloor}$.

3. Indu-Bala product

The join of graphs $\mathbb{H}_1$ and $\mathbb{H}_2$, recognized as $\mathbb{H}_1 + \mathbb{H}_2$, is the disjoint union of graphs $\mathbb{H}_1$ and $\mathbb{H}_2$ along with all the edges joining $\mathcal{V}(\mathbb{H}_1)$ and $\mathcal{V}(\mathbb{H}_2)$. Now the Indu-Bala product $\mathbb{H}_1 \blacktriangledown \mathbb{H}_2$ is constructed from two disjoint copies of $\mathbb{H}_1 + \mathbb{H}_2$ by connecting the corresponding vertices in the two copies of $\mathbb{H}_2$. The order and size of $\mathbb{H}_1 \blacktriangledown \mathbb{H}_2$ are $2(s_1 + s_2)$ and $2t_1 + 2t_2 + 2s_1s_2 + s_2$, respectively. Upcoming lemma describes the distinct properties of this product.

Lemma 2. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

(a) $$\omega_{\mathbb{H}_1 \blacktriangledown \mathbb{H}_2}(w) = \begin{cases} \omega_{\mathbb{H}_1}(w) + s_2 \deg_{\mathbb{H}_1}(w) + 2t_2 + s_2(s_1+1), & \text{if } w \in \mathcal{V}(\mathbb{H}_1), \\ \omega_{\mathbb{H}_2}(w) + (s_1+2)\deg_{\mathbb{H}_2}(w) + 2t_1 + s_1(s_2+1) + 1, & \text{if } w \in \mathcal{V}(\mathbb{H}_2). \end{cases}$$

(b) $$ecc_{\mathbb{H}_1 \blacktriangledown \mathbb{H}_2}(w) = \begin{cases} 3, & \text{if } w \in \mathcal{V}(\mathbb{H}_1), \\ 2, & \text{if } w \in \mathcal{V}(\mathbb{H}_2). \end{cases}$$

Next, we compute the eccentric adjacency index of $\mathbb{H}_1 \blacktriangledown \mathbb{H}_2$.

Theorem 2. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

$$\xi^{ad}(\mathbb{H}_1 \blacktriangledown \mathbb{H}_2) = \frac{2}{3}\mathcal{M}_1(\mathbb{H}_1) + \mathcal{M}_1(\mathbb{H}_2) + s_1s_2\left(\frac{2}{3}s_1 + s_2 + \frac{5}{3}\right) + \frac{10}{3}(s_2t_1 + s_1t_2) + 4t_2 + s_2.$$

Proof. By using Lemma 2 in (1.1), we get

$$\xi^{ad}(\mathbb{H}_1 \blacktriangledown \mathbb{H}_2) = 2\sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)} \frac{\omega_{\mathbb{H}_1}(w_1) + s_2\deg_{\mathbb{H}_1}(w_1) + 2t_2 + s_2(s_1+1)}{3} + 2\sum_{w_1 \in \mathcal{V}(\mathbb{H}_2)} \frac{\omega_{\mathbb{H}_2}(w_1) + (s_1+2)\deg_{\mathbb{H}_2}(w_1) + 2t_1 + s_1(s_2+1) + 1}{2}$$

$$\begin{aligned}
\xi^{ad}(\mathbb{H}_1 \blacktriangledown \mathbb{H}_2) &= \frac{2}{3}\left(\sum_{w_1 \in \mathcal{V}_{\mathbb{H}_1}} \omega_{\mathbb{H}_1}(w_1) + s_2\sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1) + \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)} (2t_2 + s_2(s_1+1))\right) \\
&+ \sum_{w_1 \in \mathcal{V}_{\mathbb{H}_2}} \omega_{\mathbb{H}_2}(w_1) + (s_1+2)\sum_{w_1 \in \mathcal{V}(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_1) + \sum_{w_1 \in \mathcal{V}(\mathbb{H}_2)} (2t_1 + s_1(s_2+1) + 1) \\
&= \frac{2}{3}\mathcal{M}_1(\mathbb{H}_1) + \frac{4}{3}s_2t_1 + \frac{2s_1}{3}(2t_2 + s_2(s_1+1)) + \mathcal{M}_1(\mathbb{H}_2) + 2t_2(s_1+2) \\
&+ s_2(2t_1 + s_1(s_2+1) + 1) \\
&= \frac{2}{3}\mathcal{M}_1(\mathbb{H}_1) + \mathcal{M}_1(\mathbb{H}_2) + s_1s_2\left(\frac{2}{3}s_1 + s_2 + \frac{5}{3}\right) + \frac{10}{3}(s_2t_1 + s_1t_2) + 4t_2 + s_2.
\end{aligned}$$

This finishes the proof. □

Example 4. (1) $\xi^{ad}(P_{s_1}\blacktriangledown P_{s_2}) = \frac{s_1s_2}{3}(2s_1+3s_2+25) - \frac{2}{3}s_1 + \frac{17}{3}s_2 - 14.$

(2) $\xi^{ad}(P_{s_1}\blacktriangledown C_{s_2}) = \frac{s_1s_2}{3}(2s_1+3s_2+25) + \frac{8}{3}s_1 + \frac{17}{3}s_2 - 4.$

(3) $\xi^{ad}(C_{s_1}\blacktriangledown C_{s_2}) = \frac{s_1s_2}{3}(2s_1+3s_2+25) + \frac{8}{3}s_1 + 9s_2.$

4. Cartesian product

The Cartesian product $\mathbb{H}_1\otimes\mathbb{H}_2$ of graphs $\mathbb{H}_1$ and $\mathbb{H}_2$ has $\mathcal{V}(\mathbb{H}_1\otimes\mathbb{H}_2) = \mathcal{V}(\mathbb{H}_1)\times\mathcal{V}(\mathbb{H}_2)$ and $(w_1,w_2)(w_1',w_2')$ is an edge in $\mathbb{H}_1\otimes\mathbb{H}_2$ if $w_1'=w_1$ and $w_2w_2'\in\mathcal{E}(\mathbb{H}_2)$, or $w_1w_1'\in\mathcal{E}(\mathbb{H}_1)$ and $w_2=w_2'$. First, we state the lemma that gives the properties of Cartesian product.

Lemma 3. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

(a) $\omega_{\mathbb{H}_1\otimes\mathbb{H}_2}(w_1,w_2) = \omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2\deg_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2).$

(b) $ecc_{\mathbb{H}_1\otimes\mathbb{H}_2}(w_1,w_2) = ecc_{\mathbb{H}_1}(w_1)+ecc_{\mathbb{H}_2}(w_2).$

Now, we describe the expression of eccentric adjacency index of $\mathbb{H}_1\otimes\mathbb{H}_2\cdots\otimes\mathbb{H}_m$ in the form of factor graphs.

Theorem 3. *Let $\mathbb{H}_1,\mathbb{H}_2,\ldots,\mathbb{H}_m$ be s_m-vertex graphs with size t_m, where $1\le m\le s$, respectively. Then $A\le\xi^{ad}(\mathbb{H}_1\otimes\mathbb{H}_2\cdots\otimes\mathbb{H}_m)\le B$, where*

$$A = \frac{1}{\sum\limits_{l=1}^{m} d_{\mathbb{H}_l}}\left(\sum_{l=1}^{m}\mathcal{M}_1(\mathbb{H}_l)\prod_{n=1,n\neq l}^{m}s_n + 8\sum_{l=1}^{m-1}t_l\sum_{k>l}^{m}\left(t_k\prod_{n=1,n\neq l,k}^{m}s_n\right)\right),$$

$$B = \frac{1}{\sum\limits_{l=1}^{m} r_{\mathbb{H}_l}}\left(\sum_{l=1}^{m}\mathcal{M}_1(\mathbb{H}_l)\prod_{n=1,n\neq l}^{m}s_n + 8\sum_{l=1}^{m-1}t_l\sum_{k>l}^{m}\left(t_k\prod_{n=1,n\neq l,k}^{m}s_n\right)\right).$$

Equalities hold if and only if $\mathbb{H}_1,\mathbb{H}_2,\ldots,\mathbb{H}_m$ are self-centered graphs.

Proof. By using Lemma 3 in (1.1), we obtain

$$\begin{aligned}\xi^{ad}(\mathbb{H}_1\otimes\mathbb{H}_2) &= \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\frac{\omega_{\mathbb{H}_1}(w_1)+\omega_{\mathbb{H}_2}(w_2)+2\deg_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2)}{ecc_{\mathbb{H}_1}(w_1)+ecc_{\mathbb{H}_2}(w_2)}\\ &\ge \frac{1}{d_{\mathbb{H}_1}+d_{\mathbb{H}_2}}\left(s_2\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)}\omega_{\mathbb{H}_1}(w_1)+s_1\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\omega_{\mathbb{H}_2}(w_2)\right. \qquad (4.1)\\ &\left.+2\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)}\deg_{\mathbb{H}_1}(w_1)\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}\deg_{\mathbb{H}_2}(w_2)\right)\end{aligned}$$

$$= \frac{1}{d_{\mathbb{H}_1} + d_{\mathbb{H}_2}} \left(s_2 \mathcal{M}_1(\mathbb{H}_1) + s_1 \mathcal{M}_1(\mathbb{H}_2) + 8 t_1 t_2 \right). \tag{4.2}$$

By applying induction on m. From result in (4.2), the result for $m = 2$ is valid. Let $m \geq 3$ and consider that the theorem satisfies for m. Use $\mathcal{H} = \mathbb{H}_1 \otimes \mathbb{H}_2 \cdots \otimes \mathbb{H}_m$. Then we have

$$\begin{aligned}
&\xi^{ad}(\mathbb{H}_1 \otimes \mathbb{H}_2 \cdots \otimes \mathbb{H}_m \otimes \mathbb{H}_{m+1}) \\
&= \xi^{ad}(\mathcal{H} \otimes \mathbb{H}_{m+1}) \\
&= \frac{1}{\sum\limits_{l=1}^{m} d_{\mathbb{H}_l} + d_{\mathbb{H}_{m+1}}} \left(\sum_{l=1}^{m} \mathcal{M}_1(\mathbb{H}_l) \prod_{n=1, n\neq l}^{m} s_n + 8 \sum_{l=1}^{m-1} t_i \sum_{k>l}^{m} \left(t_k \prod_{n=1, n\neq l,k}^{m} s_n \right) \right. \\
&\left. + |\mathcal{V}(\mathcal{H})| \mathcal{M}_1(\mathbb{H}_{m+1}) + 8 t_{m+1} |\mathcal{E}(\mathcal{H})| \right) \\
&= \frac{1}{\sum\limits_{l=1}^{m} d_{\mathbb{H}_l} + d_{\mathbb{H}_{m+1}}} \left(\sum_{l=1}^{m} \mathcal{M}_1(\mathbb{H}_l) \prod_{n=1, n\neq l}^{m} s_n + 8 \sum_{l=1}^{m-1} t_l \sum_{k>l}^{m} \left(t_k \prod_{n=1, n\neq l,k}^{m} s_n \right) \right. \\
&\quad \left. + (s_1 s_2 \dots s_m) \mathcal{M}_1(\mathbb{H}_{m+1}) + 8 t_{m+1} \sum_{l=1}^{m} t_l \prod_{n=1, n\neq l}^{m} s_n \right) \\
&\quad = \frac{1}{\sum\limits_{l=1}^{m+1} d_{\mathbb{H}_l}} \left(\sum_{l=1}^{m+1} \mathcal{M}_1(\mathbb{H}_l) \prod_{n=1, n\neq l}^{m+1} s_n + 8 \sum_{l=1}^{m} t_l \sum_{k>l}^{m+1} \left(t_k \prod_{n=1, n\neq l,k}^{m+1} s_n \right) \right).
\end{aligned}$$

where $|\mathcal{E}(\mathcal{H})| = \sum\limits_{l=1}^{m} t_l \prod\limits_{n=1, n\neq l}^{m} s_n$. Analogously, we can derive

$$\xi^{ad}(\mathbb{H}_1 \otimes \mathbb{H}_2 \cdots \otimes \mathbb{H}_m \otimes \mathbb{H}_{m+1}) \leq \frac{1}{\sum\limits_{l=1}^{m} r_{\mathbb{H}_l}} \left(\sum_{l=1}^{m} \mathcal{M}_1(\mathbb{H}_l) \prod_{n=1, n\neq l}^{m} s_n + 8 \sum_{l=1}^{m-1} t_l \sum_{k>l}^{m} \left(t_k \prod_{n=1, n\neq l,k}^{m} s_n \right) \right).$$

The proof is complete. □

As a conclusion of Theorem 3, we obtain an expression for the eccentric adjacency index of j-th Cartesian power of $\mathbb{H}$.

Corollary 1. *Let $\mathbb{H}$ be a s-vertex graph and j be a positive integer. Then*

$$\frac{s^{j-2}}{r_{\mathbb{H}}} \left(s \mathcal{M}_1(\mathbb{H}) + 4t^2 (j-1) \right) \geq \xi^{ad}(\mathbb{H}^j) \geq \frac{s^{j-2}}{d_{\mathbb{H}}} \left(s \mathcal{M}_1(\mathbb{H}) + 4t^2 (j-1) \right).$$

Example 5. Let $\mathcal{S} = C_{l_1} \otimes C_{l_2}$ and $\mathcal{R} = P_{l_1} \otimes C_{l_2}$, for some integers $l_1, l_2 \geq 3$, denote a C_4-nanotorus and C_4-nanotube, respectively. Then from Theorem 3 and Proposition 1, we get $\xi^{ad}(\mathcal{S}) = \dfrac{32l_1l_2}{\left\lfloor \frac{l_1}{2} \right\rfloor + \left\lfloor \frac{l_2}{2} \right\rfloor}$ and $\dfrac{l_2(8l_1-7)}{l_1 + \left\lfloor \frac{l_2}{2} \right\rfloor - 1} \leq \xi^{ad}(\mathcal{R}) \leq \dfrac{l_2(8l_1-7)}{\left\lfloor \frac{l_1}{2} \right\rfloor + \left\lfloor \frac{l_2}{2} \right\rfloor}$.

Example 6. Let $\mathbb{H} = P_a \otimes P_b$ be the rectangular grid, depicted in Fig 2. From Theorem 3 and Proposition 1, we have

$$\frac{2(8ab-7a-7b+4)}{a+b-2} \leq \xi^{ad}(\mathbb{H}) \leq \frac{2(8ab-7a-7b+4)}{\left\lfloor \frac{a}{2} \right\rfloor + \left\lfloor \frac{b}{2} \right\rfloor}.$$

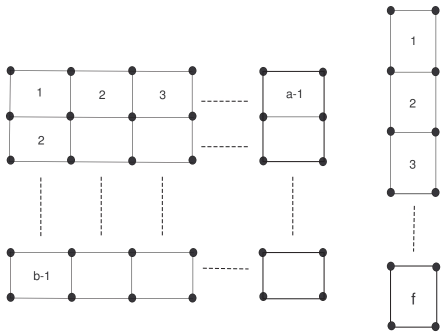

Figure 2. **The rectangular grid $P_a \otimes P_b$ and the ladder graph $P_2 \otimes P_{j+1}$.**

Example 7. The ladder graph $\mathbb{L}_j = P_2 \otimes P_{j+1}$ with $2j+2$ vertices is constructed by j squares (see Figure 2). By Theorem 3 and Proposition 1, we get

$$\frac{2(9j-1)}{j+1} \leq \xi^{ad}(\mathbb{L}_j) \leq \frac{2(9j-1)}{1 + \left\lfloor \frac{j}{2} \right\rfloor}.$$

Example 8. A connected graph with vertices in type of k-tuples $(w_1, w_2, \ldots, w_k)$ where $0 \leq w_l \leq s_l - 1$, $s_l \geq 2$, and $(w_1, w_2, \ldots, w_k)(w'_1, w'_2, \ldots, w'_k)$ is an edge if the correlated tuples differ in exactly one place, is written by $\mathbb{H}_{s_1,s_2,\ldots,s_k} = \bigotimes_{l=1}^{k} K_{s_l}$ and known as a Hamming graph. From Theorem 3 and Proposition 1, we get $\xi^{ad}(\mathbb{H}_{s_1,s_2,\ldots,s_k}) = \dfrac{1}{k}\left(\prod_{l=1}^{k} s_l\right)\left(\sum_{l=1}^{k}(s_l-1)\right)^2$.
If $s_1 = s_2 = \cdots = s_k = 2$, then the k-dimensional Hamming graph is named as a hypercube, and it is expressed by $\mathcal{Q}_k$ (shown in Fig 3). Then $\xi^{ad}(\mathcal{Q}_k) = \xi^{ad}(K_2^k) = k2^k$.

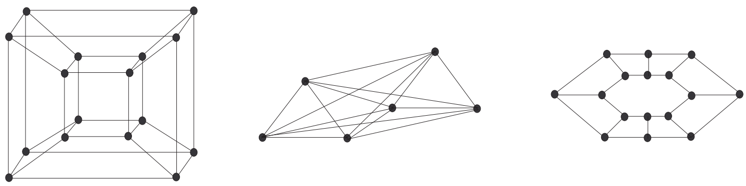

Figure 3. **The hypercube graphs $\mathcal{Q}_4$, Rook's graph of K_3 and K_2, and 8-Prism.**

Example 9. For a s-prism graph, $K_2 \otimes C_s$, the eccentric adjacency index is given by

$$\xi^{ad}(K_2 \otimes C_s) = \begin{cases} \dfrac{36s}{s+2}, & \text{if } s \text{ is even,} \\ \dfrac{36s}{s+1}, & \text{if } s \text{ is odd.} \end{cases}$$

Example 10. The Rook's graph is obtained by the Cartesian product of K_{l_1} and K_{l_2}. Then $\xi^{ad}(K_{l_1} \otimes K_{l_2}) = \dfrac{l_1 l_2(l_1 + l_2 - 2)}{2}$.

5. Disjunction

The disjunction of $\mathbb{H}_1$ and $\mathbb{H}_2$, is a graph with $\mathcal{V}(\mathbb{H}_1 \vee \mathbb{H}_2) = \mathcal{V}(\mathbb{H}_1) \times \mathcal{V}(\mathbb{H}_2)$ and $(w_1, w_2)(w_1', w_2') \in \mathcal{E}(\mathbb{H}_1 \vee \mathbb{H}_2)$ whenever $w_2 w_2' \in \mathcal{E}(\mathbb{H}_2)$ or $w_1 w_1' \in \mathcal{E}(\mathbb{H}_1)$, indicated as $\mathbb{H}_1 \vee \mathbb{H}_2$. The order of $\mathbb{H}_1 \vee \mathbb{H}_2$ is $s_1 s_2$, and size is $t_1 s_2^2 + t_2 s_1^2 - 2t_1 t_2$. First, we state following lemma in which we present some properties of disjunction of graphs.

Lemma 4. *Let $\mathbb{H}_1 \ncong K_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

(*a*) $\omega_{\mathbb{H}_1 \vee \mathbb{H}_2}(w_1, w_2) = (s_2^2 - 2t_2)\omega_{\mathbb{H}_1}(w_1) + (s_1^2 - 2t_1)\omega_{\mathbb{H}_2}(w_2) + 2(s_1 t_2 \deg_{\mathbb{H}_1}(w_1) + s_2 t_1 \deg_{\mathbb{H}_2}(w_2))$
$-s_2 \omega_{\mathbb{H}_1}(w_1) \deg_{\mathbb{H}_2}(w_2) - s_1 \omega_{\mathbb{H}_2}(w_2) \deg_{\mathbb{H}_1}(w_1) + \omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2)$.

(*b*) $ecc_{\mathbb{H}_1 \vee \mathbb{H}_2}(w_1, w_2) = \begin{cases} 1, & \textit{if } w_1 \in \Gamma(\mathbb{H}_1), w_2 \in \Gamma(\mathbb{H}_2), \\ 2, & \textit{otherwise.} \end{cases}$

In the upcoming theorem, we present the result on the eccentric adjacency index for $\mathbb{H}_1 \vee \mathbb{H}_2$.

Theorem 4. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

$$\xi^{ad}(\mathbb{H}_1 \vee \mathbb{H}_2) = \frac{1}{2}|\Gamma(\mathbb{H}_2)|(s_2 - 2t_2) \sum_{w_1 \in \Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}s_2(s_2^2 - 2t_2 - (s_2 - 1)|\Gamma(\mathbb{H}_2)|)\mathcal{M}_1(\mathbb{H}_1)$$
$$+ \frac{1}{2}|\Gamma(\mathbb{H}_1)|(s_1 - 2t_1) \sum_{w_2 \in \Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \frac{1}{2}s_1(s_1^2 - 2t_1 - (s_1 - 1)|\Gamma(\mathbb{H}_1)|)\mathcal{M}_1(\mathbb{H}_2)$$
$$+ 3|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|(s_1 t_2(s_1 - 1) + s_2 t_1(s_2 - 1)) + \frac{1}{2}(2s_1 s_2 t_2 - s_1\mathcal{M}_1(\mathbb{H}_2))$$
$$\sum_{w_2 \in \mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2) + \frac{1}{2}\left(2s_1 s_2 t_2 - s_2\mathcal{M}_1(\mathbb{H}_1)\right) \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1)$$
$$+ \frac{1}{2}\mathcal{M}_1(\mathbb{H}_1)\mathcal{M}_1(\mathbb{H}_2) + \frac{1}{2} \sum_{w_2 \in \Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1 \in \Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1).$$

Proof. By using Lemma 4 in (1.1), we obtain

$$\xi^{ad}(\mathbb{H}_1 \vee \mathbb{H}_2)$$
$$= \sum_{w_1 \in \Gamma(\mathbb{H}_1)} \sum_{w_2 \in \Gamma(\mathbb{H}_2)} ((s_2^2 - 2t_2)\omega_{\mathbb{H}_1}(w_1) + (s_1^2 - 2t_1)\omega_{\mathbb{H}_2}(w_2) + 2(s_1 t_2 \deg_{\mathbb{H}_1}(w_1)$$
$$+ s_2 t_1 \deg_{\mathbb{H}_2}(w_2)) - s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2) - s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1) + \omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2))$$
$$+ \frac{1}{2} \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \sum_{w_2 \in \Gamma(\mathbb{H}_2)} ((s_2^2 - 2t_2)\omega_{\mathbb{H}_1}(w_1) + (s_1^2 - 2t_1)\omega_{\mathbb{H}_2}(w_2) + 2(s_1 t_2 \deg_{\mathbb{H}_1}(w_1)$$
$$+ s_2 t_1 \deg_{\mathbb{H}_2}(w_2)) - s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2) - s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1) + \omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2))$$
$$+ \frac{1}{2} \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)} \sum_{w_2 \in \mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} ((s_2^2 - 2t_2)\omega_{\mathbb{H}_1}(w_1) + (s_1^2 - 2t_1)\omega_{\mathbb{H}_2}(w_2) + 2(s_1 t_2 \deg_{\mathbb{H}_1}(w_1)$$
$$+ s_2 t_1 \deg_{\mathbb{H}_2}(w_2)) - s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2) - s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1) + \omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2))$$
$$+ \frac{1}{2} \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)} \sum_{w_2 \in \mathcal{V}(\mathbb{H}_2)} ((s_2^2 - 2t_2)\omega_{\mathbb{H}_1}(w_1) + (s_1^2 - 2t_1)\omega_{\mathbb{H}_2}(w_2) + 2(s_1 m_2 \deg_{\mathbb{H}_1}(w_1)$$
$$+ s_2 t_1 \deg_{\mathbb{H}_2}(w_2)) - s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2) - s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1) + \omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2))$$
$$= \frac{1}{2}(s_2^2 - 2m_2)|\Gamma(\mathbb{H}_2)| \sum_{w_1 \in \Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}s_2(s_2^2 - 2t_2)\left(\sum_{w_1 \in \Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\right.$$
$$\left. + \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\right) + \frac{1}{2}(s_1^2 - 2t_1)|\Gamma(\mathbb{H}_1)| \sum_{w_2 \in \Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \frac{1}{2}s_1(s_1^2 - 2t_1)$$
$$\left(\sum_{w_2 \in \Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \sum_{w_2 \in \mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)\right) + |\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|(2s_1 t_2(s_1 - 1)$$
$$+ 2s_2 t_1(s_2 - 1) + s_2 t_1(s_1 - 1) + s_1 t_2(s_2 - 1)) - s_2(s_2 - 1)|\Gamma(\mathbb{H}_2)|\left(\frac{1}{2} \sum_{w_1 \in \mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\right.$$

$$+ \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\bigg) - s_1(s_1-1)|\Gamma(\mathbb{H}_1)|\bigg(\frac{1}{2}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)\bigg)$$

$$+ s_1t_2s_2 \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1) + s_2t_1s_1 \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2) - \frac{1}{2}s_2\bigg(\sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$+ \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\bigg) \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2) - \frac{1}{2}s_1\bigg(\sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)$$

$$+ \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)\bigg) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1) + \sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)\bigg(\sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$+ \frac{1}{2}\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\bigg) + \frac{1}{2}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)\bigg(\sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$+ \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)\bigg)$$

$$= \frac{1}{2}(s_2^2-2t_2)|\Gamma(\mathbb{H}_2)| \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}s_2(s_2^2-2t_2) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}(s_1^2-2t_1)|\Gamma(\mathbb{H}_1)|$$

$$\sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \frac{1}{2}s_1(s_1^2-2m_1) \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + 3|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|(s_1t_2(s_1-1)$$

$$+ s_2t_1(s_2-1)) - \frac{1}{2}s_2(s_2-1)|\Gamma(\mathbb{H}_2)| \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) - \frac{1}{2}s_2(s_2-1)|\Gamma(\mathbb{H}_2)| \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$- \frac{1}{2}s_1(s_1-1)|\Gamma(\mathbb{H}_1)| \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) - \frac{1}{2}s_1(s_1-1)|\Gamma(\mathbb{H}_1)| \sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2)$$

$$+ s_1s_2t_2 \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1) - \frac{1}{2}s_1 \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1)$$

$$+ s_1s_2t_1 \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2) - \frac{1}{2}s_2 \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) \sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2)$$

$$+ \frac{1}{2}\sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}\sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$+ \frac{1}{2}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1)$$

$$= \frac{1}{2}|\Gamma(\mathbb{H}_2)|(s_2-2t_2) \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1) + \frac{1}{2}s_2(s_2^2-2t_2-(s_2-1)|\Gamma(\mathbb{H}_2)|)\mathcal{M}_1(\mathbb{H}_1)$$

$$+ \frac{1}{2}|\Gamma(\mathbb{H}_1)|(s_1-2t_1) \sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) + \frac{1}{2}s_1(s_1^2-2t_1-(s_1-1)|\Gamma(\mathbb{H}_1)|)\mathcal{M}_1(\mathbb{H}_2)$$

$$+ 3|\Gamma(\mathbb{H}_1)||\Gamma(\mathbb{H}_2)|(s_1t_2(s_1-1)+s_2t_1(s_2-1)) + \frac{1}{2}(2s_1s_2t_2 - s_1\mathcal{M}_1(\mathbb{H}_2))$$

$$\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)\setminus\Gamma(\mathbb{H}_2)} \deg_{\mathbb{H}_2}(w_2) + \frac{1}{2}\left(2s_1s_2t_2 - s_2\mathcal{M}_1(\mathbb{H}_1)\right) \sum_{w_1\in\mathcal{V}(\mathbb{H}_1)\setminus\Gamma(\mathbb{H}_1)} \deg_{\mathbb{H}_1}(w_1)$$

$$+ \frac{1}{2}\mathcal{M}_1(\mathbb{H}_1)\mathcal{M}_1(\mathbb{H}_2) + \frac{1}{2}\sum_{w_2\in\Gamma(\mathbb{H}_2)} \omega_{\mathbb{H}_2}(w_2) \sum_{w_1\in\Gamma(\mathbb{H}_1)} \omega_{\mathbb{H}_1}(w_1).$$

This completes the proof. □

Example 11. (1) $\xi^{ad}(K_{s_1}\vee K_{s_2})=s_1s_2^2(s_1-1)^2(2-s_2)+s_1^2s_2(s_2-1)^2(2-s_1)+s_1s_2(s_1-1)(s_2-1)(4s_1s_2-s_1-s_2+1)$.

(2) $$\xi^{ad}(C_{s_1}\vee C_{s_2})=\begin{cases}2s_1s_2(s_1+s_2)^2-8s_1s_2(s_1+s_2+1), & \text{if } s_1\geq 4 \text{ and } s_2\geq 4,\\ 6(s_2^3+s_2^2-2s_2-3), & \text{if } s_1=3 \text{ and } s_2\geq 4,\\ 6(s_1^3+s_1^2-2s_1-3), & \text{if } s_2=3 \text{ and } s_1\geq 4,\\ 996, & \text{if } s_1=3 \text{ and } s_2=3.\end{cases}$$

6. Symmetric difference

The symmetric difference $\mathbb{H}_1\oplus\mathbb{H}_2$ of $\mathbb{H}_1$ and $\mathbb{H}_2$, is a graph with $\mathcal{V}(\mathbb{H}_1\oplus\mathbb{H}_2)=\mathcal{V}(\mathbb{H}_1)\times\mathcal{V}(\mathbb{H}_2)$ and $(w_1,w_2)(w_1',w_2')\in\mathcal{E}(\mathbb{H}_1\oplus\mathbb{H}_2)$ whenever $w_2w_2'\in\mathcal{E}(\mathbb{H}_2)$ or $w_1w_1'\in\mathcal{E}(\mathbb{H}_1)$ but not both. The order of $\mathbb{H}_1\oplus\mathbb{H}_2$ is s_1s_2, and size is $t_1s_2^2+t_2s_1^2-4t_1t_2$. Now, we present certain features of the symmetric difference of graphs.

Lemma 5. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

(a) $\omega_{\mathbb{H}_1\oplus H_2}(w_1,w_2) = (s_2^2-2t_2)\omega_{\mathbb{H}_1}(w_1)+(s_1^2-2t_1)\omega_{\mathbb{H}_2}(w_2)+2(s_1t_2\deg_{\mathbb{H}_1}(w_1)+s_2t_1\deg_{\mathbb{H}_2}(w_2))$
$-2s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2)-2s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1)+2\omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2)$.

(b) $ecc_{\mathbb{H}_1\vee\mathbb{H}_2}(w_1,w_2)=2$.

Now, we present the expression of the eccentric adjacency index of graphs.

Theorem 5. *Let $\mathbb{H}_1$ and $\mathbb{H}_2$ be s_1-vertex and s_2-vertex graphs of size t_1 and t_2, respectively. Then*

$$\xi^{ad}(\mathbb{H}_1\oplus\mathbb{H}_2)=\frac{1}{2}(s_2(s_2^2-6t_2)\mathcal{M}_1(\mathbb{H}_1)+s_1(s_1^2-6t_1)\mathcal{M}_1(\mathbb{H}_2))+4s_1s_2t_1t_2+\mathcal{M}_1(\mathbb{H}_1)\mathcal{M}_1(\mathbb{H}_2).$$

Proof. By using Lemma 5 in (1.1), we obtain

$$\begin{aligned}&\xi^{ad}(\mathbb{H}_1\oplus\mathbb{H}_2)\\&=\frac{1}{2}\sum_{w_1\in\mathcal{V}(\mathbb{H}_1)}\sum_{w_2\in\mathcal{V}(\mathbb{H}_2)}((s_2^2-2t_2)\omega_{\mathbb{H}_1}(w_1)+(s_1^2-2t_1)\omega_{\mathbb{H}_2}(w_2)+2(s_1t_2\deg_{\mathbb{H}_1}(w_1)\\&+s_2t_1\deg_{\mathbb{H}_2}(w_2))-2s_2\omega_{\mathbb{H}_1}(w_1)\deg_{\mathbb{H}_2}(w_2)-2s_1\omega_{\mathbb{H}_2}(w_2)\deg_{\mathbb{H}_1}(w_1)+2\omega_{\mathbb{H}_1}(w_1)\omega_{\mathbb{H}_2}(w_2))\\&=\frac{1}{2}(s_2(s_2^2-2t_2)\mathcal{M}_1(\mathbb{H}_1)+s_1(s_1^2-2t_1)\mathcal{M}_1(\mathbb{H}_2)+4s_1t_2s_2t_1+4s_2t_1s_1t_2-4s_2t_2\mathcal{M}_1(\mathbb{H}_1)\\&-4s_1t_1\mathcal{M}_1(\mathbb{H}_2)+2\mathcal{M}_1(\mathbb{H}_1)\mathcal{M}_1(\mathbb{H}_2))\\&=\frac{1}{2}(s_2(s_2^2-6t_2)\mathcal{M}_1(\mathbb{H}_1)+s_1(s_1^2-6t_1)\mathcal{M}_1(\mathbb{H}_2))+4s_1s_2t_1t_2+\mathcal{M}_1(\mathbb{H}_1)\mathcal{M}_1(\mathbb{H}_2).\end{aligned}$$

This completes the proof. □

Example 12. (1) $\xi^{ad}(P_{s_1} \oplus P_{s_2}) = 2s_1s_2(s_1+s_2)^2 - 3(s_1^3+s_2^3 - 8s_1s_2(s_2+2s_1+1) + 18(s_1^2+s_2^2+21(s_1+s_2) - 36$.

(2) $\xi^{ad}(P_{s_1} \oplus C_{s_2}) = 2s_1s_2(s_1+s_2)^2 - 3s_2^2(s_2-6) + 24s_2 - 4s_1s_2(4s_2+3s_1+1)$.

(3) $\xi^{ad}(C_{s_1} \oplus C_{s_2}) = 2s_1s_2(s_1+s_2)(s_1+s_2-6s_1s_2) - 16s_1s_2$.

7. Chemical Relevance

Topological indices have become increasingly accessible, with their numbers continuously expanding. Many of these indices are derived purely through mathematical approaches, often overlooking their chemical significance. To bridge this gap, a set of practical guidelines has been established to assist in selecting an appropriate molecular descriptor from numerous available options. One key criterion is the ability to predict molecular properties and behaviors. To assess the predictive capability of topological indices, researchers commonly conduct quantitative structure-property relationship (QSPR) analyses, which compare theoretical attributes with experimental data from specific reference compounds. Randić and Trinajstić [39] proposed using octanes as benchmark data for the initial evaluation of invariants. Our findings indicate that the ξ^{ad} index exhibits a strong correlation with entropy (S), acentric factor (AF) and standard enthalpy of vaporization ($DHVAP$) of octanes. The performance of the ξ^{ad} index is examined using the following regression equation:

$$P = mT + c, \tag{7.1}$$

where P represents the studied property, m denotes the slope, T stands for the topological index, and c is the intercept. The regression analysis also includes supplementary parameters such as the standard error (SE), F-test value (F), and significance F (SF) to ensure a thorough evaluation. Additionally, the coefficient of determination (r^2) is utilized, with r indicating the correlation coefficient. For ξ^{ad}, the relationship in equation (7.1) can be expressed as follows:

$$S = -1.716\,\xi^{ad} + 120.968,$$
$$r^2 = 0.877, \quad SE = 1.632, \quad F = 114.287, \quad SF = 1.08 \times 10^{-8}. \tag{7.2}$$

$$AF = -0.014\,\xi^{ad} + 0.463,$$
$$r^2 = 0.958, \quad SE = 0.007, \quad F = 369.339, \quad SF = 1.77 \times 10^{-12}. \tag{7.3}$$

$$DHVAP = -0.129\,\xi^{ad} + 10.299,$$
$$r^2 = 0.693, \quad SE = 0.219, \quad F = 36.176, \quad SF = 1.8 \times 10^{-5}. \tag{7.4}$$

The linear fittings of ξ^{ad} with S, AF and $DHVAP$ for octanes are depicted in Figure 4.

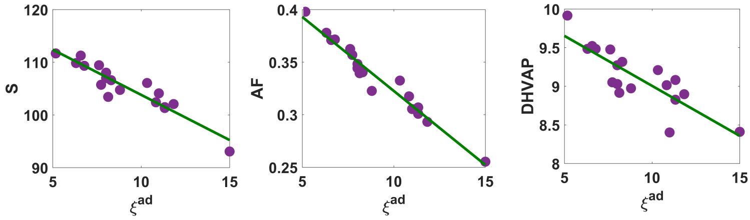

Figure 4. **Linear fitting of ξ^{ad} with S, AF and $DHVAP$ for octanes.**

8. Conclusion

The analysis of networks and graphs with their structural characteristics is a very massive topics of research with developing noteworthiness. The greatest and well known method in the development of structural properties is quantitative calculations, that encode the structural information of any graphical structure by a number. We have determined the formulas related to the eccentric adjacency index of composition, Indu-Bala product, Cartesian product, disjunction and symmetric difference of graphs and implement these outcomes for certain significant classes of chemical structures in the form of co-factors of graph operations, in this research paper. We have observed in relations (7.2), (7.3), (7.4), that the ξ^{ad} index correlates well with the entropy, acentric factor and standard enthalpy of vaporization for octanes.

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