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Research Article

Degree distance index of class of graphs

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Abstract: The topological indices are the numerical parameters of a graph that characterize the topology of a graph and are usually graph invariant. The topological indices are classified based on the properties of graphs. The degree distance index is the topological index which is calculated by counting the degrees and distance between the vertices. In this paper, the degree distance index of the connected thorn graph, the graph obtained by joining an edge between two connected graphs, and one vertex union of two connected graphs are calculated.

Keywords: topological index, chemical graphs, degree of a vertex, distance between two vertices.

AMS Subject classification: 05C07, 05C12, 05C90

1. Introduction

Let G be a simple connected graph. The distance between two vertices v_i and v_j of the connected graph G is the number of edges in the shortest $v_i - v_j$ path. The degree of a vertex v_i is the number of edges incident with the vertex v_i. The status of a vertex v_i in any graph is the sum of the distance from v_i to all other vertices of a graph and it is denoted by $\sigma_G(v_i)$. That is, the status of the vertex v_i in the graph G is $\sigma_G(v_i) = \sum_{v_j \in V(G)} d_G(v_i, v_j)$. A thorn graph of a graph G of order n with p_i parameters denoted by G_P, is a graph obtained by adding p_i- pendant edges to the vertex v_i of G, where p_i are the non-negative integers and $1 \leq i \leq n$. The one vertex

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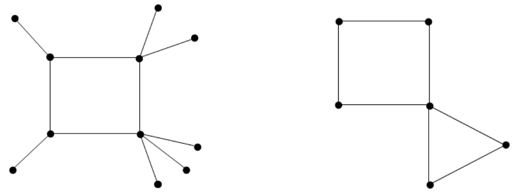

Figure 1. Thorn graph of cycle C_4 with 1,2,3,1- parameters and the one vertex union of cycles C_4 and C_3.

union of k graphs is obtained by identifying one vertex of all the k graphs. For all the terminologies refer [3, 9].

The topological indices are the numerical parameters associated with the graph which are usually graph invariant. The topological index of a graph is based on the properties of graphs such as degree, distance, number of non-incident edges and so on. From this index it is possible to analyze the mathematical values and further investigate some physicochemical properties of a molecule. Therefore, it is also called a molecular descriptor. The Wiener index was introduced by H. Wiener [10] is the first topological index to be used in Chemistry to determine the boiling point of paraffin. Let $d_G(v_i, v_j)$ be the distance between the vertices v_i and v_j and $deg_G(v_i)$ be the degree of the vertex v_i. Then the Wiener index of a graph G is,

$$W(G) = \sum_{1 \le i < j \le n} d(v_i, v_j).$$

The degree distance index was introduced by A. A. Dobrynin and A. A. Kochetova [4] which correlates degree distance with Wiener index. The degree distance of a connected graph G is defined as,

$$DD(G) = \sum_{1 \le i < j \le n} (deg(v_i) + deg(v_j))\, d(v_i, v_j).$$

The Wiener index of thorn graphs can be found in [5]. The Schultz index of thorn graphs is obtained in [8]. Also, the Gutman index of thorn graphs can be seen in [2]. Motivated by these studies, in this article, the degree distance index of the thorn graph of a connected graph with p_i-parameters is obtained. The degree distance indices of few product graphs are calculated in [1, 6, 7]. This motivated us to find the degree distance index of a graph obtained by joining an edge between two connected graphs and one vertex union of two connected graphs.

From the obtained results, the degree distance indices of thorn paths, caterpillars, thorn rings, thorn stars, Kragujevac trees, and dendrimers can be obtained by substituting desired parameters. The degree distance indices of the kite graph and lollipop graph can also be obtained by using the results in section 2. Where the kite graph and lollipop graph are few graphs obtained by both joining an edge between two graphs and one vertex union of two graphs. Also, there are many chemical compounds whose graphical representations are isomorphic to the graphs considered in this study. For example,

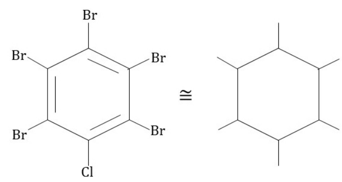

Figure 2. $1,2,3,4,5$ **pentabromo-6-chloro benzene.**

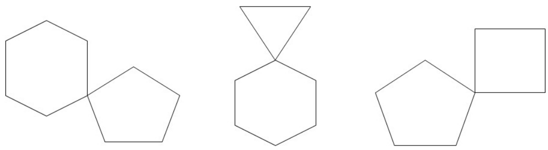

Figure 3. **Spiro[5,4]decane, spiro[5,2]octane and spiro[4,3]octane.**

2. Main results

In this section, degree distance indices of thorn graph with p_i- parameters, the graph obtained by joining an edge between two graphs, and one vertex union of two graphs are calculated.

Theorem 1. *Let G be a connected graph with vertex set $\{v_1, v_2, \ldots, v_n\}$ and let $p_1, p_2, \ldots, p_n$ be the pendent edges on the vertices $v_1, v_2, \ldots, v_n$ respectively. Then the degree distance index of thorn graph G_P with p_i parameters of a graph G is*

$$\begin{aligned} DD(G_P) = {} & DD(G) + (n+2m-2)\sum_{i=1}^{n} p_i + 2\sum_{i=1}^{n} p_i^2 + \left(\sum_{i=1}^{n} p_i\right)^2 + 4\sum_{1\le i<j\le n} p_i p_j \\ & + \sum_{1\le i<j\le n} (2p_i + 2p_j + 4p_i p_j + p_j \deg_G(v_i) + p_i \deg_G(v_j)) d_G(v_i, v_j). \end{aligned}$$

Proof. Let G_P be the thorn graph of a connected graph G with p_i, $1 \le i \le n$, parameters and let $\{v_1, v_2, \ldots, v_n\}$ be vertices of G and p_{ik}, $1 \le i \le n$, be pendant vertices adjacent to v_i, where k is any non-negative integer.

$$\begin{aligned} DD(v_i, v_l) &= \sum_{1\le i<j\le n} (\deg_G(v_i) + p_i + \deg_G(v_j) + p_j) d_G(v_i, v_j) \\ &= DD(G) + \sum_{1\le i<j\le n} (p_i + p_j) d_G(v_i, v_j). \end{aligned}$$

$$DD(v_i, p_{ik}) = \sum_{\substack{1\le i\le n,\\ 1\le k\le p_i}} (\deg_G(v_i) + p_i + \deg_{G_P}(p_{ik}))d_{G_P}(v_i, p_{ik})$$
$$= \sum_{i=1}^{n}(p_i \deg_G(v_i) + p_i^2 + p_i). \text{ since } d_{G_P}(v_i, p_{ik}) = 1 \text{ and } \deg_{G_P}(p_{ik}) = 1$$

$$DD(p_{ik}, p_{il}) = \sum_{\substack{1\le i\le n,\\ 1\le k<l\le p_i}} (\deg_{G_P}(p_{ik}) + \deg_{G_P}(p_{il}))d_{G_P}(p_{ik}, p_{il})$$
$$= \sum_{i=1}^{n}(1+1)(2)\binom{p_i}{2} = 2\left(\sum_{i=1}^{n} p_i^2 - \sum_{i=1}^{n} p_i\right).$$

$$DD(v_i, p_{jk}) = \sum_{\substack{1\le i<j\le n,\\ 1\le k\le p_i}} (deg_G(v_i) + p_i + deg_{G_P}(p_{jk}))d_{G_P}(v_i, p_{jk})$$
$$= \sum_{1\le i<j\le n} (deg_G(v_i) + p_i + 1)(d_G(v_i, v_j) + 1)$$
$$= \sum_{1\le i<j\le n} (deg_G(v_i) + p_i + 1)d_G(v_i, v_j) + \sum_{1\le i<j\le n} (deg_G(v_i) + p_i + 1).$$

But,

$$\sum_{1\le i<j\le n} (deg_G(v_i) + p_i + 1)d_G(v_i, v_j) = \sum_{1\le i<j\le n} (p_i + p_j + 2p_ip_j + p_j \deg_G(v_i) + p_i \deg_G(v_j))d_G(v_i, v_j),$$

and

$$\sum_{1\le i<j\le n} (deg_G(v_i) + p_i + 1) = 2m\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i \deg_G(v_i) + \left(\sum_{i=1}^{n} p_i\right)^2 - \sum_{i=1}^{n} p_i^2 + n\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i$$
$$= (2m+n-2)\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i \deg_G(v_i) + \left(\sum_{i=1}^{n} p_i\right)^2 - \sum_{i=1}^{n} p_i^2.$$

Therefore,

$$DD(v_i, p_{jk}) = \sum_{1\le i<j\le n} (p_i + p_j + 2p_ip_j + p_j \deg_G(v_i) + p_i \deg_G(v_j))d_G(v_i, v_j)$$
$$+ (2m+n-2)\sum_{i=1}^{n} p_i - \sum_{i=1}^{n} p_i \deg_G(v_i) + \left(\sum_{i=1}^{n} p_i\right)^2 - \sum_{i=1}^{n} p_i^2.$$

$$DD(p_{ik}, p_{jl}) = \sum_{\substack{1\le i<j\le n \\ 1\le k\le p_i \\ 1\le l\le p_j}} (deg_{G_P}(p_{ik}) + deg_{G_P}(p_{jl}))d_{G_P}(p_{ik}, p_{jl})$$
$$= \sum_{1\le i<j\le n} p_i p_j (1+1)(d_G(v_i, v_j) + 2) = 2\sum_{1\le i<j\le n} p_i p_j d_G(v_i, v_j) + 4\sum_{1\le i<j\le n} p_i p_j.$$

Combining all the degree distances, we get

$$DD(G_P) = DD(G) + (n+2m-2)\sum_{i=1}^{n} p_i + 2\sum_{i=1}^{n} p_i^2 + \left(\sum_{i=1}^{n} p_i\right)^2 + 4\sum_{1\le i<j\le n} p_i p_j$$
$$+ \sum_{1\le i<j\le n} (2p_i + 2p_j + 4p_i p_j + p_j \deg_G(v_i) + p_i \deg_G(v_j))d_G(v_i, v_j).$$

□

Corollary 1. *If G_p is a thorn graph of a connected graph G of order n with p parameters on every vertex of G, then $DD(G_p) = (p+1)DD(G) + 4p(p+1)W(G) + (n+3np+2m-2)np$.*

Proof. By substituting $\sum_{i=1}^{n} p_i = np$, $\sum_{i=1}^{n} p_i^2 = np^2$, $\left(\sum_{i=1}^{n} p_i\right)^2 = n^2p^2$, $\sum_{1\le i<j\le n} p_i p_j = \binom{n}{2}p^2$, and $\sum_{1\le i<j\le n} (p_j \deg_g(v_i) + p_i \deg_G(v_j)) = pDD(G)$ in Theorem 1, we get the result. □

Theorem 2. *Let $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ be two connected graphs and G' be a graph obtained by joining an edge u_1v_1 between G_1 and G_2, where $u_1 \in V(G_1)$ and $v_1 \in V(G_2)$. Then,*

$$DD(G') = DD(G_1) + DD(G_2) + 2(m_2+1)\sigma_{G_1}(u_1) + 2(m_1+1)\sigma_{G_2}(v_1)$$
$$+ n_1\left(2m_2 + 1 + \sum_{j=1}^{n_2} \deg_{G_2}(v_j)d_{G_2}(v_1, v_j)\right)$$
$$+ n_2\left(2m_1 + 1 + \sum_{i=1}^{n_1} \deg_{G_1}(u_i)d_{G_1}(u_1, u_i)\right),$$

where, $\sigma_{G_1}(u_1)$, $\sigma_{G_2}(v_1)$ are the status of the vertices u_1, v_1 respectively.

Proof. Let G_1 and G_2 be two connected graphs with vertex set $\{u_1, u_2, \ldots, u_{n_1}\}$ and $\{v_1, v_2, \ldots, v_{n_2}\}$ respectively. Let the new edge be joined between the vertices u_1 and v_1.

$$
\begin{aligned}
DD(u_i,u_j) &= \sum_{j=2}^{n_1}(\deg_{G_1}(u_1)+1+\deg_{G_1}(u_j))d_{G_1}(u_1,u_j)\\
&+ \sum_{2\le i<j\le n_1}(\deg_{G_1}(u_i)+\deg_{G_1}(u_j))d_{G_1}(u_i,u_j)\\
&= DD(G_1)+\sum_{i=2}^{n_1} d_{G_1}(u_1,u_j)\\
&= DD(G_1)+\sigma_{G_1}(u_1).
\end{aligned}
$$

$$
\begin{aligned}
DD(v_i,v_j) &= \sum_{i=2}^{n_2}(\deg_{G_2}(v_1)+1+\deg_{G_2}(v_j))d_{G_2}(v_1,v_j)\\
&+ \sum_{2\le i<j\le n_2}(\deg_{G_2}(v_i)+\deg_{G_2}(v_j))d_{G_2}(v_i,v_j)\\
&= DD(G_2)+\sum_{i=2}^{n_2} d_{G_2}(v_1,v_j)\\
&= DD(G_2)+\sigma_{G_2}(v_1).
\end{aligned}
$$

$$
DD(u_1,v_1) = deg_{G_1}(u_1)+deg_{G_2}(v_1)+2.
$$

$$
\begin{aligned}
DD(u_1,v_j) &= \sum_{j=2}^{n_2}(deg_{G_1}(u_1)+1+deg_{G_2}(v_j))(d_{G_2}(v_1,v_j)+1)\\
&= deg_{G_1}(u_1)\sigma_{G_2}(v_1)+(n_2-1)deg_{G_1}(u_1)+\sigma_{G_2}(v_1)+n_2-1\\
&+ 2m_2-\deg_{G_2} v_1+\sum_{j=2}^{n_2}\deg_{G_2}(v_j).
\end{aligned}
$$

$$
\begin{aligned}
DD(v_1,u_i) &= \sum_{i=2}^{n_1}(deg_{G_2}(v_1)+1+deg_{G_1}(u_i))(d(u_1,u_i)+1)\\
&= deg_{G_2}(v_1)\sigma_{G_1}(u_1)+(n_1-1)deg_{G_2}(v_1)+\sigma_{G_1}(u_1)+n_1-1\\
&+ 2m_1-\deg_{G_1} u_1+\sum_{i=2}^{n_1}\deg_{G_1}(u_i).
\end{aligned}
$$

$$
DD(u_i,v_j) = \sum_{j=2}^{n_2}\sum_{i=2}^{n_1}(deg_{G_1}(u_i)+deg_{G_2}(v_j))(d_{G_1}(u_1,u_i)+d_{G_2}(v_1,v_j)+1),
$$

where,

$$
\sum_{j=2}^{n_2}\sum_{i=2}^{n_1}(deg_{G_1}(u_i)+deg_{G_2}(v_j)) = (n_2-1)(2m_1-\deg_{G_1}(u_1))+(n_1-1)(2m_2-\deg_{G_2}(v_1)),
$$

$$\sum_{j=2}^{n_2}\sum_{i=2}^{n_1}(deg_{G_1}(u_i)d_{G_1}(u_1,u_i)+deg_{G_2}(v_j)d_{G_2}(v_1,v_j)) = (n_2-1)\sum_{i=2}^{n_1}(deg_{G_1}(u_i)d_{G_1}(u_1,u_i) + (n_1-1)\sum_{j=2}^{n_2}(deg_{G_2}(v_j)d_{G_2}(v_1,v_j),$$

and

$$\sum_{j=2}^{n_2}\sum_{i=2}^{n_1}(deg_{G_1}(u_i)d_{G_2}(v_1,v_j)+deg_{G_2}(v_j)d_{G_1}(u_1,u_i)) = (2m_1-deg_{G_1}(u_1))\sigma_{G_2}(v_1) + (2m_2-deg_{G_2}(v_1))\sigma_{G_1}(u_1).$$

Therefore,

$$\begin{aligned}DD(G') &= DD(G_1)+DD(G_2)+2(m_2+1)\sigma_{G_1}(u_1)+2(m_1+1)\sigma_{G_2}(v_1)\\ &+n_1\left(2m_2+1+\sum_{j=1}^{n_2}\deg_{G_2}(v_j)d_{G_2}(v_1,v_j)\right)\\ &+n_2\left(2m_1+1+\sum_{i=1}^{n_1}\deg_{G_1}(u_i)d_{G_1}(u_1,u_i)\right).\end{aligned}$$

□

Corollary 2. *Let $G_1(n_1,m_1)$ and $G_2(n_2,m_2)$ be two connected regular graphs with regularity r_1 and r_2 respectively and G' be a graph obtained by joining an edge u_1v_1 between G_1 and G_2. Then*

$$\begin{aligned}DD(G') &= DD(G_1)+DD(G_2)+2(m_2+1)\sigma_{G_1}(u_1)+2(m_1+1)\sigma_{G_2}(v_1)\\ &+n_1(2m_2+1+r_2\sigma_{G_2}(v_1))+n_2(2m_1+1+r_1\sigma_{G_1}(u_1)),\end{aligned}$$

where, $\sigma_{G_1}(u_1)$, $\sigma_{G_2}(v_1)$ are the status of the vertices u_1, v_1 respectively.

Proof. The proof of the corollary can be obtained by substituting

$$\sum_{i=2}^{n_1}\deg_G(u_i)d_{G_1}(u_1,u_i) = r_1\sum_{i=2}^{n_1}d_{G_1}(u_1,u_i) = r_1\sigma_{G_1}(u_1)$$

and

$$\sum_{j=2}^{n_2}\deg_G(v_j)d_{G_2}(v_1,v_j) = r_2\sum_{j=2}^{n_2}d_{G_2}(v_1,v_j) = r_1\sigma_{G_2}(v_1)$$

in Theorem 2. □

Theorem 3. *Let G'' be a graph obtained by one vertex union of two connected graphs $G_1(n_1,m_1)$ and $G_2(n_2,m_2)$ and the common vertices of G_1 and G_2 be u_1 and v_1 respectively. Let $\sigma_{G_1}(u_1)$, $\sigma_{G_2}(v_1)$ be the statues of the vertices u_1 and v_1 respectively. Then,*

$$\begin{aligned}DD(G'') &= DD(G_1)+DD(G_2)+2m_1\sigma_{G_2}(v_1)+2m_2\sigma_{G_1}(u_1)\\ &+(n_2-1)\sum_{i=2}^{n_1}\deg_{G_1}(u_1)d_{G_1}(u_1,u_i)+(n_1-1)\sum_{j=2}^{n_2}\deg_{G_2}(v_1)d_{G_2}(v_1,v_j).\end{aligned}$$

Proof. Let G_1 and G_2 be two connected graphs with vertex set $\{u_1, u_2, \ldots, u_{n_1}\}$ and $\{v_1, v_2, \ldots, v_{n_2}\}$ respectively. Let u_1, v_1 be the common vertices of G_1 and G_2. Then,

$$\begin{aligned}DD(u_i, u_j) &= \sum_{i=2}^{n}(\deg_{G_1}(u_1) + \deg_{G_2}(v_1) + \deg_{G_1} u_i)d_{G_1}(u_i, u_j)\\ &+ \sum_{2\le i<j\le n}(\deg_{G_1}(u_i) + \deg_{G_1}(u_j))d_{G_1}(u_i, u_j)\\ &= DD(G_1) + \deg_{G_2}(v_1)\sigma(u_1).\end{aligned}$$

$$\begin{aligned}DD(v_i, v_j) &= \sum_{i=2}^{n}(\deg_{G_2}(v_1) + \deg_{G_1}(u_1) + \deg_{G_2}(v_i))d_{G_2}(v_i, v_j)\\ &+ \sum_{2\le i<j\le n}(\deg_{G_2}(v_i) + \deg_{G_2}(v_j))d_{G_2}(v_i, v_j)\\ &= DD(G_2) + \deg_{G_1}(u_1)\sigma(v_1).\end{aligned}$$

$$\begin{aligned}DD(u_i, v_j) &= \sum_{i,j=2}^{n}(\deg_{G_1}(u_i) + \deg_{G_2}(v_j))(d_{G_1}(u_1, u_i) + d_{G_2}(v_1, v_j))\\ &= \sum_{i,j=2}^{n}\deg_{G_1}(u_i)d_{G_2}(v_1, v_j) + \sum_{i,j=2}^{n}\deg_{G_2}(v_j)d_{G_1}(u_1, u_i)\\ &+ (n_2 - 1)\sum_{j=2}^{n}\deg_{G_1}(u_i)d_{G_1}(u_1, u_i) + (n_1 - 1)\sum_{i,j=2}^{n}\deg_{G_2}(v_j)d_{G_2}(v_1, v_j)\\ &= (2m_1 - \deg_{G_1}(u_1))\sigma_{G_2}(v_1) + (2m_1 - \deg_{G_1}(u_1))\sigma_{G_2}(v_1)\\ &+ (n_2 - 1)\sum_{j=2}^{n}\deg_{G_1}(u_i)d_{G_1}(u_1, u_i) + (n_1 - 1)\sum_{i,j=2}^{n}\deg_{G_2}(v_j)d_{G_2}(v_1, v_j).\end{aligned}$$

Therefore,

$$\begin{aligned}DD(G'') &= DD(G_1) + DD(G_2) + 2m_1\sigma_{G_2}(v_1) + 2m_2\sigma_{G_1}(u_1)\\ &+ (n_2 - 1)\sum_{i=2}^{n_1}\deg_{G_1}(u_1)d_{G_1}(u_1, u_i) + (n_1 - 1)\sum_{j=2}^{n_2}\deg_{G_2}(v_1)d_{G_2}(v_1, v_j).\end{aligned}$$

□

Corollary 3. *Let G'' be a graph obtained by one vertex union of two connected regular graphs $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ with regularity r_1 and r_2 respectively and the common vertices of G_1 and G_2 be u_1 and v_1 respectively. Then,*

$$\begin{aligned}DD(G'') &= DD(G_1) + DD(G_2) + (2m_1 + r_2(n_1 - 1))\sigma_{G_2}(v_1)\\ &+ (2m_2 + r_1(n_2 - 1))\sigma_{G_1}(u_1),\end{aligned}$$

where, $\sigma_{G_1}(u_1)$, $\sigma_{G_2}(v_1)$ are the statues of the vertices u_1 and v_1 respectively.

Proof. Proof follows by substituting,

$$\sum_{i=2}^{n} \deg_{G_1}(u_1) d_{G_1}(u_1, u_i) = r_1 \sigma_{G_1}(u_1)$$

and

$$\sum_{j=2}^{n} \deg_{G_2}(v_1) d_{G_2}(v_1, v_j) = r_2 \sigma_{G_2}(v_1)$$

in Theorem 3. □

3. Conclusion

In this study, the authors obtained the degree distance index of thorn graph of a connected graph with p_i- parameters, a graph obtained by joining an edge between two connected graphs, and one vertex union of two connected graphs. From the results obtained in Section 2, degree distance indices of different thorn graphs, kite graph, lollipop graph, and other classes of graphs which are isomorphic to the graph joined by an edge between two graphs and one vertex union of two graphs can be obtained.

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