Communications in Combinatorics and Optimization
Vol. 9, No. 2 (2024), pp. 317-327
https://doi.org/10.22049/cco.2023.27744.1337

CCO
Commun. Comb. Optim.

Short Note

Power dominator chromatic numbers of splitting graphs of certain classes of graphs

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Received: 31 March 2022; Accepted: 18 January 2023
Published Online: 22 January 2023

Abstract: Domination in graphs and coloring of graphs are two main areas of investigation in graph theory. Power domination is a variant of domination in graphs introduced in the study of the problem of monitoring an electric power system. Based on the notions of power domination and coloring of a graph, the concept of power dominator coloring of a graph was introduced. The minimum number of colors required for power dominator coloring of a graph G is called the power dominator chromatic number $\chi_{pd}(G)$ of G, which has been computed for some classes of graphs. Here we compute the power dominator chromatic number for splitting graphs of certain classes of graphs.

Keywords: Splitting Graphs, Power domination, Coloring

AMS Subject classification: 05C15, 05C69

1. Introduction

The theory of domination in graphs [6, 7] is an important and extensively investigated topic of research in graph theory having applications in different areas. A graph $G = (V, E)$ is a mathematical structure with a finite set of elements, called vertices and a finite set of pairs of vertices, called edges. We consider finite undirected graphs without loops and multiple edges. Given a graph $G = (V, E)$, a subset S of V is

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a dominating set of G if every vertex in $V - S$ has at least one neighbour in S. There are several variants of the notion of domination in graphs. Haynes et al. [5] developed the concept of power domination while formulating in graph theoretical terms, the problem of monitoring an electric power system by placing as few phase measurement units (PMUs) in the system as possible.

In a graph $G = (V, E)$ representing an electric power system, a vertex represents an electrical node and an edge represents a transmission line joining two electrical nodes. A set S of vertices is defined to be a power dominating set of a graph G if every vertex and every edge in the system is "monitored" by the set S (following a set of rules for power system monitoring). The minimum cardinality of a power dominating set of a graph is its power domination number.

On the other hand graph coloring [1] has been an intensively investigated area of study in graph theory. Several variations of the notion of graph coloring have been introduced and investigated by many researchers. A proper coloring [1] of a graph G is an assignment of colors to the vertices of G with the property that no two adjacent vertices receive the same color. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G.

Based on the concept of domination in graphs, the notion of dominator coloring in a graph has been introduced and investigated [4]. A dominator coloring [4] of G is a proper coloring of G in which every vertex of G dominates every vertex of at least one color class with the convention that if $\{v\}$ is a color class, then v dominates the color class $\{v\}$. In other words the vertex is either adjacent to all the vertices of one color class or is the only vertex in its color class, by which it will dominate its own color class. Given a graph G, while the chromatic number $\chi(G)$ represents the minimum number of colors required to color the vertices of G so that no two adjacent vertices receive the same color, the dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G which has been found for many classes of graphs.

Based on the concepts of coloring and power domination, a new variant of coloring called power dominator coloring of a graph G was introduced in [10] which we recall here in a more precise form. For a vertex u in a graph G, we associate a monitoring set $M(u)$ [3] as follows:

Step (i) : $M(u) = N[u]$, the closed neighbourhood of u

Step (ii) : add a vertex w to $M(u)$, (which is originally not in $M(u)$) whenever w has a neighbour $v \in M(u)$ such that all the neighbours of v other than w, are already in $M(u)$

Step (iii) : repeat *Step* (ii) until no other vertex could be added to $M(u)$.

Then we say that u power dominates the vertices in $M(u)$. The power dominator coloring of G is a proper coloring of G such that every vertex of the vertex set V power dominates all vertices of at least one color class. The power dominator chromatic number $\chi_{pd}(G)$ is the minimum number of colors required for a power dominator coloring of G. This has been computed for certain classes of graphs such as corona graphs, degree splitting graphs. centipede graphs, barbell graphs, crown

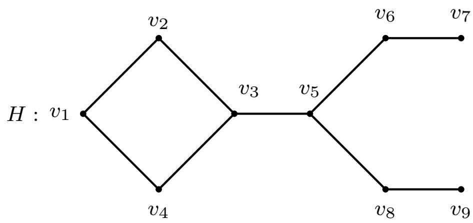

Figure 1. **A Graph H with $\chi_{pd}(H) = 3$**

graphs, sunlet graphs, tadpole graphs, spider graphs, book graphs, lollipop graphs and Kragujevac trees [10–13, 15].

We illustrate with an example.

Example 1. For the graph H in Figure 1, the power dominator chromatic number $\chi_{pd}(H) = 3$. In fact, a proper coloring of the vertices can be done as follows: Assign color 1 to the vertex v_5, color 2 to the vertices v_1, v_3, v_6, v_8 and color 3 to the vertices v_2, v_4, v_7, v_9. For the vertex v_1, initially the monitoring set $M(v_1) = \{v_1, v_2, v_4\}$ using the *Step*(i) in the formation of monitoring set. Now the vertex v_3 is the only vertex, which is not in $M(v_1)$, but adjacent to v_2 and hence v_3 is added to $M(v_1)$ using *Step*(ii) in the formation of monitoring set. Continuing the process, the vertex v_5 is the only vertex adjacent to v_3 but not in $M(v_1)$ and hence is added to $M(v_1)$. No other vertex could be added to $M(v_1)$. Thus $M(v_1) = \{v_1, v_2, v_3, v_4, v_5\}$. In fact each of the vertices v_2, v_3, v_4 has the same monitoring set as that of v_1. Hence each of the vertices v_1, v_2, v_3, v_4 power dominates the vertex v_5 and so color class with color 1 as v_5 is the only vertex with color 1. Note that none of these vertices v_1, v_2, v_3, v_4 can power dominate the color class with color 2 or with color 3. Likewise each of the vertices v_6, v_7, v_8, v_9 also power dominates the color class with color 1. The vertex v_5 power dominates its own color class. Hence each vertex power dominates all the vertices in at least one color class. It can be seen that we cannot obtain a power dominator coloring in this graph with just two colors. Thus $\chi_{pd}(G) = 3$. Note also that the dominator chromatic number $\chi_d(G)$ for this graph is 5.

The splitting graph of a graph G was introduced by Sampathkumar and Walikar [9]. Let G be a (p, q) graph. The splitting graph $S(G)$ of G is obtained as follows: For each vertex v of G, a new vertex v' is introduced and joined to all those vertices of G which are adjacent to v. Observe that $S(G)$ is a $(2p, 3q)$ graph, as the construction of $S(G)$ introduces $2q$ new edges, in addition to the edges of G. A graph G and its splitting graph $S(G)$ are shown in Figure. 2. Some domination parameters of the splitting graph of a graph have been studied in [2].

In the following sections we obtain the power dominator chromatic numbers of splitting graphs of different classes of graphs.

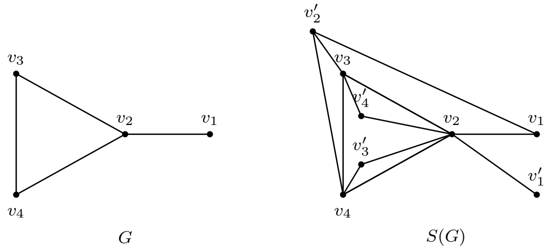

Figure 2. **A graph G and its splitting graph $S(G)$**

2. Preliminaries

For standard notions on graphs and for unexplained concepts we refer to [1, 16]. A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. The complete graph on n vertices is denoted by K_n. A complete bipartite graph with a bipartition (V_1, V_2) of the vertex set, is denoted by $K_{m,n}$ where V_1 has m vertices and V_2 has n vertices. A star graph (or simply, a star) is a complete bipartite graph with $m = 1$ and it is denoted by $K_{1,n}$.

Definition 1. The graph bistar, denoted by $B_{n,n}$, is a K_2 with n pendant edges attached at each end point.

Definition 2. A wheel graph $W_{1,n}$ is a graph obtained by joining a vertex u, called the apex vertex, to all the vertices of a cycle C_n and the vertices of the cycle are called the rim vertices of the wheel.

Definition 3. The Helm graph H_n with $n \geq 1$, is defined to be the graph obtained from a wheel graph $W_{1,n}$ by attaching a new pendant vertex at each vertex of the n-cycle.

Definition 4. The n-sunlet graph S_n is a graph on $2n$ vertices with a cycle C_n and each vertex of the cycle being joined to a new pendant vertex.

We recall some known results.

Theorem 1. *[10] For any graph G, $\chi_{pd}(G) \leq \chi_d(G)$.*

Theorem 2. *[10] (i) For a path $P_n, n \geq 2$ on n vertices, $\chi_{pd}(P_n) = 2$.*
(ii) For a cycle C_n, $n \geq 3$,

$$\chi_{pd}(C_n) = \begin{cases} 2, & \text{if } n, \text{ is even} \\ 3, & \text{if } n, \text{ is odd.} \end{cases}$$

Theorem 3. *[10] (i) For the complete bipartite graph $K_{m,n}$, $\chi_{pd}(K_{m,n}) = 2$.*
(ii) If G is a connected graph of order n, then $\chi_{pd}(G) = n$ if and only if $G = K_n$.

3. Power Dominator Coloring of Splitting graphs of Some Classes of Graphs

In this section we obtain the power dominator chromatic numbers of splitting graphs of some standard classes of graphs. We first consider path P_n on n vertices. We begin with an example.

Example 2. Figure 3 shows a power dominator coloring of the splitting graph $S(P_5)$ of path P_5. The colors given to vertices are shown (enclosed in parantheses) near the vertices. The number of colors used is 4 and it can be seen that this is a minimum number. In fact if a proper coloring of the path P_5 (which requires only two colors) is done with colors 1 and 2, then the primed vertices in the splitting graph $S(P_5)$ cannot power dominate any color class unless all the primed vertices are given different colors. This will only increase the number of colors used.

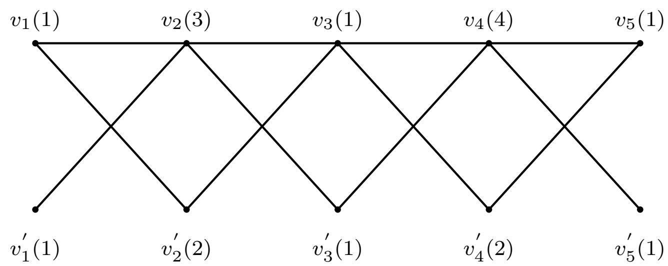

Figure 3. **Power dominator coloring of splitting graph $S(P_5)$ of path P_5**

Remark 1. We first compute the power dominator chromatic number of the splitting graph of path P_n for small values of n with $n = 3, 4, 5$ and 6. It can be verified that the proper coloring described below (in these cases) is such that every vertex in $S(P_n), n = 3, 4, 5, 6$ power dominates at least one color class and that no lesser number of colors will yield a power dominator coloring. The vertices of path P_n are denoted by v_i while the vertex in the splitting graph corresponding to v_i is denoted by v_i' which is adjacent to the neighbours of v_i, $i = 1, \cdots n$.
When $n = 3$, the vertices v_1 and v_3 are colored by color 1 and v_2 is colored by 3 while $v_i', i = 1, 2, 3$ are colored by color 2. The vertex v_2' power dominates color class 1 as well as 3 while all other vertices power dominate color class 3. In this case the power dominator chromatic number is 3.
When $n = 4$ the vertices v_1 and v_4 are colored by color 2. The vertices v_2, v_3 are colored by colors 3 and 4 respectively. The vertices $v_i', i = 1, 2, 3, 4$ are colored by color 1. The vertices v_1 and v_1' power dominate the color class 3. The vertices v_4 and v_4' power dominate the color

class 4. The vertices v_2 and v'_2 power dominate the color classes 3 and 4. In this case the power dominator chromatic number is 4.
When $n = 5$ the vertices v_1, v_3, v_5 are colored by color 2. The vertices v_2, v_4 are colored by colors 3 and 4 respectively. The vertices $v'_i, i = 1, 2, 3, 4, 5$ are colored by color 1. In this case also the power dominator chromatic number is 4.
When $n = 6$ the vertices v_1, v_3 are colored by color 1; v_4, v_6 are colored by color 2 and the vertices v_2 and v_5 are colored by colors 3 and 4 respectively. The vertices $v'_i, i = 1, 2, 3, 4, 5, 6$ are alternately colored by colors 1 and 2 starting with color 1 for v'_1. Again in this case the power dominator chromatic number is 4.

Theorem 4. *For a path P_n of order $n \geq 7$,*

$$\chi_{pd}(S(P_n)) = \begin{cases} \frac{n}{2} + 1, & \text{when } n = 4k + 6,\ k \geq 1 \\ \lceil \frac{n+1}{2} \rceil + 1, & \text{otherwise .} \end{cases}$$

Proof. Consider a path $P_n, n \geq 7$ with n vertices $v_1, v_2, v_3, \cdots, v_n$ and $n - 1$ edges. In constructing the splitting graph $S(P_n)$ of the path P_n, new vertices $v'_1, v'_2, \cdots, v'_n$ are introduced and for all i, $1 \leq i \leq n$, the vertex v'_i is made adjacent to the neighbours of v_i in P_n. The graph $S(P_n)$ consists of $2n$ vertices and $3(n - 1)$ edges. By the definition of power domination the vertex v'_1 power dominates only v_2 while v'_n power dominates only v_{n-1}. The vertex v'_2 power dominates v_1, v_2, v_3 and v'_{n-1} power dominates v_{n-2}, v_{n-1}, v_n and for each i, $3 \leq i \leq n - 2$, the vertex v'_i power dominates v_{i-1} and v_{i+1}.
A power dominator coloring of $S(P_n)$ with a minimum number of colors is obtained as described below: In order to use a minimum number of colors, the idea employed is that the primed vertices are given a proper coloring with just two colors and each of these vertices is made to power dominate at least a color class of vertices in the path P_n. But then at least 2 colors are required for the vertices of P_n and thus at least 4 colors are needed to obtain a power dominator coloring of $S(P_n)$ except for $n = 3$.
The vertices v'_i, $1 \leq i \leq n$ are assigned colors 1 and 2 alternately with v'_1 assigned the color 1. A proper coloring of vertices v_i, $1 \leq i \leq n$, is done as follows:

In each case we describe a proper coloring of vertices which yields a power dominator coloring with a minimum number of colors : The vertices v'_i, for $1 \leq i \leq n$ are assigned colors 1 and 2 alternately beginning with color 1 for v'_1.

Case 1. Let $n = 4k + 3$, $k \geq 1$.
In this case the vertex v_2 is colored by a unique color 3 and hence v_2 power dominates itself. The vertices v_{4m+1} and v_{4m+2}, $1 \leq m \leq \frac{n-3}{4}$ are colored by distinct colors $2m + 3$ and $2m + 4$ respectively and the number of colors used for these vertices is $2 \times \frac{n-3}{4}$. Each of these vertices v_{4m+1} and v_{4m+2} power dominates its own color class. For the vertices among v_i, $1 \leq i \leq n$ that remain to be colored, assign the color of the corresponding vertices v'_i. We thus obtain a power dominator coloring of $S(P_n)$, with every vertex power dominating at least one color class, as desired. The total number of colors used is $3 + 2 \times \frac{n-3}{4} = 2 + \frac{n-1}{2} = \lceil \frac{n+1}{2} \rceil + 1$.

The remaining Cases 2, 3 and 4 respectively correspond to $n = 4k + 4, k \geq 1$, $n = 4k + 5, k \geq 1$ and $n = 4k + 6, k \geq 1$. In each of these cases, the vertices v_2 and v_{n-1} are colored by unique colors 3 and 4 respectively. The vertices v_{4m+1} and v_{4m+2} are colored by distinct colors $4m+1$ and $4m+2$ respectively, with $1 \leq m \leq \frac{n-4}{4}$ in Case 2, $1 \leq m \leq \frac{n-5}{4}$ in Case 3 and $1 \leq m \leq \frac{n-6}{4}$ in Case 4. The number of colors used for these vertices is $2 \times \frac{n-4}{4}, n \geq 8$ in Case 2, $2 \times \frac{n-5}{4}, n \geq 9$ in Case 3 and $2 \times \frac{n-6}{4}, n \geq 10$ in Case 4. In all these three cases, each of these vertices v_{4m+1} and v_{4m+2} power dominates its own color class as in case 1. Again for the vertices among v_i, $1 \leq i \leq n$ that remain to be colored, assign the color of the corresponding vertices v'_i. Note that the vertices v_1, v'_1, v'_2, v_3 and v'_3 power dominate the color class 3 and $v_n, v'_n, v'_{n-1}\ v_{n-2}$ and v'_{n-2} power dominate the color class 4. The vertices v_i and v'_i, $4 \leq i \leq n-3$ power dominate any one of the color classes $4m+1$ or $4m+2$. We thus obtain, in each of these cases, a power dominator coloring of $S(P_n)$, with every vertex power dominating at least one color class, as desired. The total number of colors used is (i) $4+2\times\frac{n-4}{4} = 1+\frac{n+2}{2}, n = 8, 12, \ldots$ in Case 2, (ii) $4+2\times\frac{n-5}{4} = 1+\frac{n+1}{2}, n = 9, 13, \ldots,$ in Case 3 and $(iii) = 4+2\times\frac{n-6}{4} = 1+\frac{n}{2}, n = 10, 14, \ldots,$ in Case 4. Hence the total number of colors used is $\frac{n}{2}+1$ in Case 4 and $\lceil\frac{n+1}{2}\rceil+1$ in all other cases. This proves the theorem. □

A result analogous to Theorem 4 is now given for the splitting graph of a cycle C_n. We first compute the power dominator chromatic number of the splitting graph of cycle C_n for small values of n with $n = 3, 4, 5$ and 6. It can be verified that the proper coloring described below (in these cases) is such that every vertex in $S(C_n), n = 3, 4, 5, 6$ power dominates at least one color class and that no lesser number of colors will yield a power dominator coloring.

When $n = 3$, the vertices v_1, v_2 and v_3 are colored by colors 1,2 and 3 respectively. The vertices v'_i, $1 \leq i \leq 3$ are assigned by the color 4. In this case $\chi_{pd}(S(C_3)) = 4$.

When $n = 4$, the vertices v'_i, $1 \leq i \leq 4$ are assigned colors 1 and 2 alternately with v'_1 assigned the color 1. The vertices v_1, v_4 are colored by colors 3 and 4 respectively and vertices v_2 and v_4 are colored by the colors of the corresponding vertices v'_2 and v'_2. In this case $\chi_{pd}(S(C_4)) = 4$.

When $n = 5$, The vertices v'_i, $1 \leq i \leq 4$ are assigned colors 1 and 2 alternately with v'_1 assigned the color 1. The vertices v_1, v_4 and v_5 are colored by colors 3, 4 and 5 respectively and vertices v_2, v_3 are colored by the colors of the corresponding primed vertices. In this case $\chi_{pd}(S(C_5)) = 5$. When $n = 6$, the vertices v_1 is colored by color 3 while v_2 and v_6 are colored by color 4. The vertices v_3 and v_5 are colored by color 1 and v_4 is colored by color 2. The vertices v'_i, $1 \leq i \leq 6$ are assigned the color 5. In this case also $\chi_{pd}(S(C_6)) = 5$.

We now state the result on the power dominator chromatic number of the splitting graph of C_n for $n \geq 7$. The proof of this result is on lines similar to the Theorem 4.

Theorem 5. *For a cycle C_n of order $n \geq 7$,*

$$\chi_{pd}(S(C_n)) = \begin{cases} \frac{n}{2} + 2, & \text{when } n = 4k+4,\ k \geq 1 \\ \lceil \frac{n+1}{2} \rceil + 2, & \text{otherwise} . \end{cases}$$

Theorem 6. *For the complete graph K_n of order $n \geq 3$,*

$$\chi_{pd}(S(K_n)) = n + 1.$$

Proof. Let the vertex set of the complete graph K_n of order $n \geq 3$ be $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. The splitting graph $S(K_n)$ is obtained by adding a new vertex v_i' corresponding to each vertex v_i, $1 \leq i \leq n$ such that v_i' is adjacent to the neighbours of v_i in K_n.
A power dominator coloring of $S(K_n)$ with minimum number of colors is obtained as follows: Assign color i to the vertex v_i for $1 \leq i \leq n$ and color all the vertices v_i', $1 \leq i \leq n$, with a new color $n+1$. Each vertex v_i power dominates itself (in addition to dominating all the remaining vertices $v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$). The vertex v_i', $1 \leq i \leq n$ power dominates at least one of the color classes i, $1 \leq i \leq n$. Since every vertex in a complete graph is adjacent to every other vertex, it is clear that we require $n+1$ colors to obtain a power dominator coloring of $S(K_n)$.Hence $\chi_{pd}(S(K_n)) = n + 1$. □

Theorem 7. *For the complete bipartite graph $K_{m,n}$ of order $m, n \geq 2$, $\chi_{pd}(S(K_{m,n})) = 3$. In particular $\chi_{pd}(S(K_{1,1})) = 2$.*

Proof. Let $V_1 = \{v_i \mid 1 \leq i \leq m\}$ and $V_2 = \{u_j \mid 1 \leq j \leq n\}$ be the vertex partition of $K_{m,n}$. Every vertex in V_1 is adjacent only to every vertex in V_2. A power dominator coloring of $S(K_{m,n})$ is as follows:
In splitting graph $S(K_{m,n})$ of the complete bipartite graph $K_{m,n}$, new vertices v_i' are added corresponding to v_i, $1 \leq i \leq m$, and each v_i' is joined to all the vertices u_j, $1 \leq j \leq n$. Likewise, new vertices u_j' are added corresponding to u_j, $1 \leq j \leq n$, and each u_j' is joined to all the vertices v_i, $1 \leq i \leq m$. The graph $S(K_{m,n})$ has $2(m+n)$ vertices and $3mn$ edges. Assign color 1 to the vertices v_i, $1 \leq i \leq m$, and color 2 to the vertices of u_j, $1 \leq j \leq n$. The remaining vertices v_i', $1 \leq i \leq m$, and u_j', $1 \leq j \leq n$, are colored by color 3. Each primed vertex power dominates either color class 1 or 2. Each vertex v_i power dominates color class 2 and u_j power dominates color class 1. Thus only three colors are needed for a power dominator coloring of $S(K_{m,n})$. But a power dominator coloring of $S(K_{m,n})$ is not possible with only two colors as already two colors are needed for the vertices of the bipartition of $K_{m,n}$. Hence $\chi_{pd}(S(K_{m,n})) = 3$. When $m = n = 1$, clearly, $\chi_{pd}(S(K_{1,1})) = 2$. □

Remark 2. It follows from Theorem 7 that for the path K_2, $\chi_{pd}(S(K_2)) = 2$ as $K_{1,1}$ is simply the path K_2.

Corollary 1. *For splitting graph of star $S(K_{1,n})$, $\chi_{pd}(S(K_{1,n})) = 3$.*

Theorem 8. *The power dominator chromatic number of splitting graph of bistar is $\chi_{pd}(S(B_{n,n})) = 4$ for $n \geq 2$.*

Proof. Consider the bistar $B_{n,n}$ with vertex set $\{v, u, v_i, u_i \mid 1 \leq i \leq n\}$ where v_i, u_i, $1 \leq i \leq n$, are the pendant vertices. In order to obtain $S(B_{n,n})$ we add v', u', v'_i, u'_i respectively corresponding to v, u, v_i, u_i where $1 \leq i \leq n$. The splitting graph of bistar $B_{n,n}$ has $4n + 4$ vertices and $6n + 3$ edges. The vertices v and u power dominate all the vertices of $S(B_{n,n})$. Color the vertices u and v by colors 1 and 2 respectively. In order to obtain power dominator coloring, we assign a new color 3 to the vertices v' and u'. The vertices $v'_i, u'_i, 1 \leq i \leq n$, are colored by the color 4. Clearly this is a power dominator coloring. Thus $\chi_{pd}(S(B_{n,n})) = 4$. □

4. Power dominator coloring of splitting graphs of some special types of graphs

Splitting graphs of some special types of graphs, namely, wheel graph, Helm graph, $n-$sunlet graph are considered and the power dominator chromatic numbers of these splitting graphs are obtained.

Theorem 9. *The power dominator chromatic number of the splitting graph of wheel graph $W_{1,n}$, $n \geq 3$, is*

$$\chi_{pd}(S(W_{1,n})) = \begin{cases} 4, & \text{if } n \text{ is even} \\ 5, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. For the wheel graph $W_{1,n}$ with $n \geq 3$, let the apex vertex be v_1 and the vertices on the rim be $v_2, v_3, \ldots, v_{n+1}$. The splitting graph $S(W_{1,n})$ of the wheel graph $W_{1,n}$ is obtained by adding to each vertex v_i, $1 \leq i \leq n+1$, of the graph $W_{1,n}$, a new vertex v'_i such that v'_i is adjacent to the neighbours of v_i in $W_{1,n}$. A power dominator coloring of $S(W_{1,n})$ is obtained by coloring the apex vertex by 1.
When n is odd, color the rim vertices $v_2, v_3, \ldots, v_n$, except v_{n+1} alternately by 2 and 3 starting with color 2 for v_2. In this case, the vertex v_n receives color 3. Color the vertex v_{n+1} by color 4. The vertices v'_i, $1 \leq i \leq n+1$, are colored by a new color 5. Hence we require 5 colors (and clearly, no less) to color $S(W_{1,n})$, when n is odd.
When n is even, as before, color the rim vertices $v_2, v_3, \ldots, v_{n+1}$, alternately by 2 and 3. The vertex v_n receives color 2 and the vertex v_{n+1} receives color 3. The vertices v'_i, $1 \leq i \leq n+1$, are colored by a new color 4. Therefore we require only 4 colors (and again, no less) to color $S(W_{1,n})$, when n is even.
In both the cases it can be see that each of the vertices power dominates at least one color class. Hence $\chi_{pd}(S(W_{1,n})), n \geq 3$ is as stated in the theorem. □

Theorem 10. *The power dominator chromatic number of splitting graph of Helm graph H_n, $n \geq 4$ is $\chi_{pd}(S(H_n)) = n+2$.*

Proof. By the definition of Helm graph, H_n is obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle. Let $V(H_n) = \{v_1\} \cup V_1 \cup V_2$ where v_1 is the apex vertex, $V_1 = \{v_i \mid 2 \leq i \leq n+1\}$ is the set of vertices on the n-cycle and $V_2 = \{v_i \mid n+2 \leq i \leq 2n+1\}$ is the set of pendant vertices incident with n-cycle such that v_{n+i} is adjacent with v_i, $2 \leq i \leq n+1$. The splitting graph of Helm graph, denoted by $S(H_n)$, is obtained by adding corresponding to each vertex v_i in H_n, a new vertex $v_i^{'}$ such that $v_i^{'}$ is adjacent to the neighbours of v_i in H_n. Since v_1, v_{n+i} and $v_{n+i}^{'}$, $2 \leq i \leq n+1$, are non-adjacent, assign color 1 to these vertices. The vertices $v_i^{'}$, $2 \leq i \leq n+1$, are colored by color 2. The remaining vertices v_i, on the n-cycle are colored by $i+1$, $2 \leq i \leq n+1$. In $S(H_n)$, each vertex v_i, $2 \leq i \leq n+1$ power dominates itself. Since v_1, v_{n+i} and v'_{n+i}, $2 \leq i \leq n+1$, are adjacent to v_i these vertices dominate the color class i. The vertex $v_i^{'}$, $2 \leq i \leq n+1$ is adjacent to the neighbours of v_i and therefore it power dominates at least one class i. So the given procedure gives a power dominator coloring of $S(H_n)$. Hence $\chi_{pd}(S(H_n)) = n+2$. □

Theorem 11. *For n-sunlet graph SL_n, $n \geq 3$, $\chi_{pd}(S(SL_n)) = n+2$.*

Proof. Let the vertices in cycle C_n of the n-sunlet graph SL_n be $w_1, w_2, \cdots, w_n$ and the remaining n pendant vertices be $v_1, v_2, \ldots, v_n$ with v_i adjacent to w_i. In order to obtain splitting graph of $S(SL_n)$ add w'_i and v'_i corresponding to w_i and v_i for $1 \leq i \leq n$. The vertex w'_i, $1 \leq i \leq n$, is adjacent to the neighbours of w_i and the vertex v'_i is adjacent to the neighbours of v_i in SL_n. Note that the vertex v'_i is not adjacent to $v_i, 1 \leq i \leq n$. So assign color 1 to all the vertices v_i and v'_i, $1 \leq i \leq n$. In order to have power dominator coloring, assign a new color $n+2$ to vertices w'_i, $1 \leq i \leq n$. The vertices v_i and v'_i power dominate the color class w_i, $1 \leq i \leq n$. Each vertex w_i, $1 \leq i \leq n$, receiving color $i+1$ power dominates itself. The vertex w'_i, $1 \leq i \leq n$, power dominates at least one of the color classes of w_j, $1 \leq j \leq n$. Hence $\chi_{pd}(SL_n) = n+1+1 = n+2$. □

5. Conclusion

The power dominator chromatic numbers of splitting graphs of certain kinds of graphs are computed. It will be interesting to find power dominator chromatic numbers of splitting graphs of other classes of graphs. The concepts of domination and coloring in Fuzzy graphs have been studied [8, 14]. The applicability of the notion of power domination and power dominator coloring can be examined for these fuzzy graphs.

Conflict of interest. The authors declare that they have no conflict of interest.

Data Availability. Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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