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Research Article

Uniqueness of rectangularly dualizable graphs

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Abstract: A generic rectangular partition is a partition of a rectangle into a finite number of rectangles provided that no four of them meet at a point. A graph $\mathcal{H}$ is called dual of a plane graph $\mathcal{G}$ if there is one−to−one correspondence between the vertices of $\mathcal{G}$ and the regions of $\mathcal{H}$, and two vertices of $\mathcal{G}$ are adjacent if and only if the corresponding regions of $\mathcal{H}$ are adjacent. A plane graph is a rectangularly dualizable graph if its dual can be embedded as a rectangular partition. A rectangular dual $\mathcal{R}$ of a plane graph $\mathcal{G}$ is a partition of a rectangle into $n-$rectangles such that (i) no four rectangles of $\mathcal{R}$ meet at a point, (ii) rectangles in $\mathcal{R}$ are mapped to vertices of $\mathcal{G}$, and (iii) two rectangles in $\mathcal{R}$ share a common boundary segment if and only if the corresponding vertices are adjacent in $\mathcal{G}$. In this paper, we derive a necessary and sufficient for a rectangularly dualizable graph $\mathcal{G}$ to admit a unique rectangular dual upto combinatorial equivalence. Further we show that $\mathcal{G}$ always admits a slicible as well as an area−universal rectangular dual.

Keywords: plane graphs, rectangularly dualizable graphs, rectangular duals, rectangular partitions

AMS Subject classification: 68R10, 68U05

1. Introduction

A *generic rectangular partition* is a partition of a rectangle into a finite number of rectangles provided that no four of them meet at a point. In this paper, we consider simple and finite graphs and a rectangular partition, we mean a *generic rectangular partition.* A graph is *simple* if it is has no multiple edges (parallel edges) as well as loops. A graph is called *planar* if it can be drawn in the Euclidean plane without

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crossing its edges except at its endpoints. A *plane graph* is a planar graph with a fixed planar drawing. It splits the Euclidean plane into connected regions called *faces*; the unbounded region is the *exterior face* (the outermost face) and all other faces are interior faces. The vertices lying on the exterior face are exterior vertices and all other vertices are the *interior vertices*. The edges of the exterior face is called the exterior edges or boundary edges. A graph $\mathcal{H}$ is called *dual* of a plane graph $\mathcal{G}$ if there is a one−to−one correspondence between the vertices of $\mathcal{G}$ and the regions of $\mathcal{H}$, and two vertices of $\mathcal{G}$ are adjacent if and only if the corresponding regions of $\mathcal{H}$ are adjacent. An *extended graph* (4−completion) E($\mathcal{G}$) of an RDG $\mathcal{G}$ is obtained by inserting a cycle of length 4 at the exterior of the RDG and then connecting the vertices of the cycle to the exterior vertices of $\mathcal{G}$. A plane graph is a *rectangularly dualizable graph* (RDG) if its dual can be embedded as a *rectangular partition*. A *rectangular dual* $\mathcal{R}$ of a plane graph $\mathcal{G}$ is a partition of a rectangle into n−rectangles such that (i) no four rectangles of $\mathcal{R}$ meet at a point, (ii) rectangles in $\mathcal{R}$ are mapped to vertices of $\mathcal{G}$, and (iii) two rectangles in $\mathcal{R}$ share a common boundary segment if and only if the corresponding vertices are adjacent in $\mathcal{G}$.
A rectangular dual R naturally induces a labeling of its extended dual graph E(G). If two rectangles of R share a vertical segment, then blue color is assigned to the corresponding edge in E(G) and is directed from left to right otherwise if they share a horizontal segment, red color is assigned to the corresponding edge in E(G) and is directed from bottom to top (see Fig. 1). Then the orientations of all edges incident to some vertex v_i of an RDG G is a clockwise sequence of these edges composed of four subsequences: vertical edges directed into v_i, followed by horizontal edges directed into v_i and then vertical and horizontal from v_i. Such labeling is called *regular edge labeling* and v_i is called a *well formed vertex*. If each vertex of $\mathcal{G}$ is well formed, then $\mathcal{G}$ is called a *well formed graph* (*an oriented graph*).

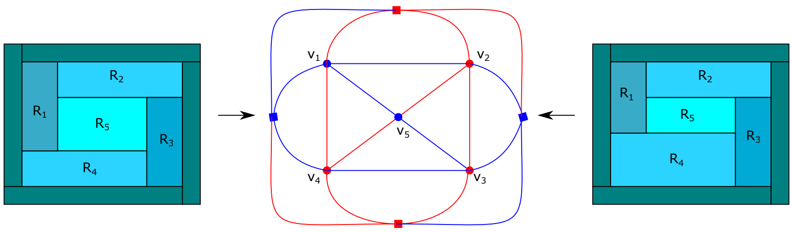

Figure 1. Two combinatorially equivalent rectangular duals inducing the same regular edge labeling of their extended graph.

Two rectangular duals $\mathcal{F}_1$ and $\mathcal{F}_2$ of the same extended graph E($\mathcal{G}$) are said to be *combinatorially equivalent* (topologically equivalent) if they induce the same regular edge labeling of E($\mathcal{G}$). Otherwise, if they induce different regular edge labelings of E($\mathcal{G}$), they are said to be topologically distinct. For instance, the rectangular duals shown in Fig. 2a and 2c are topologically distinct since they induce different regular edge labelings (shown in Fig. 2d and 2e respectively) of their extended dual graph shown in Fig. 2b. In fact, the edge between v_{10} and v_4 has different color in their respective regular edge labelings. On the other hand, the rectangular duals shown in Fig. 1 are combinatorially equivalent since they induce the same regular edge labelings; the only difference is that the dimensions of the component rectangles are different.

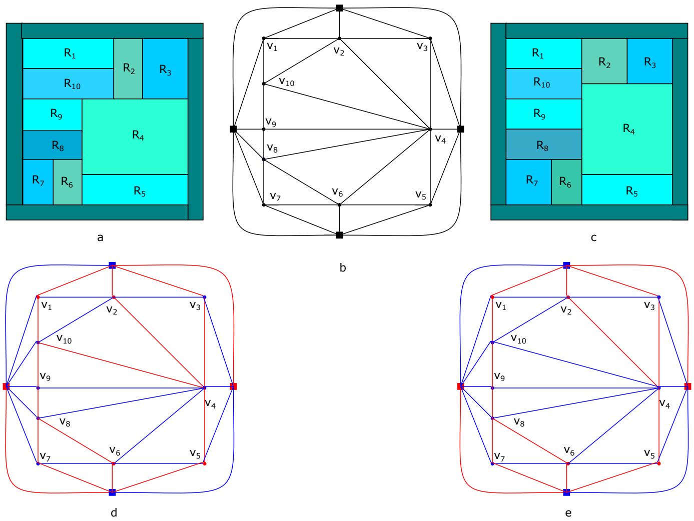

Figure 2. Two topologically distinct rectangular duals (a & c) corresponding an extended RDG (b) induce distinct regular edge labelings (d & e).

1.1. Related Work

Not every plane graph can be rectangularly dualized [11, 14, 16]. Constructive rectangular duals of planar graphs can be seen in [2, 9, 10, 23].
In general, an RDG may admit many rectangular duals. Rectangular duals of a given RDG thus generated are adjacency preserving. Adjacency preserving transformations of rectangular duals have been studied using graph notion [12, 13, 20]. By these transformations, a number of topologically distinct rectangular duals of a given RDG can be generated. Such transformations generate different regular edge labelings of an extended RDG representing a given rectangular dual. Any arbitrary regular edge labeling of an RDG may not guarantee to admit a rectangular dual. More precisely, an RDG, in general, admits a lot of regular edge labelings. In 1997, Kant and He [10] presented an algorithm for deriving a regular edge labeling obtained. Buchin et al. [4] established an upper bound on the number of edge regular labelings of an RDG. The concept of regular edge labelings is not only important because of their connection to find topological distinct rectangular partitions but also because of their connection to 4−connected plane graphs. Biedl et al. [3] showed that 4−connected plane graphs with at least four vertices on the exterior face can be extended to an irreducible triangulation. Regular edge labelings can then be used to obtain straight−line drawings of

these graphs on a small grid [7]. Fusy [7] showed that there is a function $\alpha : V \to Z$ (the set of integers) such that the regular edge labelings of an irreducible triangulation G are in bijection with the $\alpha-$orientations of the angular graph of G.
Besides the notion of strong equivalence of rectangular partitions, there is also a notion of weak equivalence, where two rectangular partitions are said to be equivalent if the incidence structure among rectangles and maximal line segments is the same. The number of weak equivalence classes can be seen in [1].

1.2. Motivation

Recently, a series of papers [1, 8, 15, 18, 19, 21, 22] have studied partitions of a rectangle into $n-$rectangles without considering prior adjacencies of its rectangles. It is evident that these enumerations produce a large solution set. In fact, they do not enumerate all rectangular partitions of a given adjacency set (or a given RDG) which can be seen in [12, 13, 20], but find all possible combinations of adjacencies among its rectangles. This makes the solution set very large. For practicality of a solution, it is computationally expensive to pick a suitable candidate solution from this large solution set.
Our motivation to find all RDGs that admit unique rectangular duals upto combinatorial equivalence stems from slicibility and area$-$universality characteristics of the unique rectangular duals upto combinatorial equivalence. It is often hard to find whether an area$-$universal assignment to a set of rectangles that corresponds to a given graph is feasible and to construct the corresponding rectangular partition. But it is easy to do with slicible rectangular duals. In an area$-$universal rectangular dual, the assignments of areas to its rectangles can be specified at later design stages. Thus, we see that the ability of finding an area$-$universal rectangular dual at the early design stage will greatly simplify the design process at later stages. Thus in VLSI circuit and architectural floorplanning, these rectangular duals are always desirable.

1.3. Preliminaries

Let $\mathcal{G}$ admit a rectangular dual $\mathcal{F}$. A corner rectangle $\mathcal{R}_c$ in $\mathcal{F}$ has two adjacent sides adjacent to the unbounded face and the vertex that is dual to $\mathcal{R}_c$ is a corner vertex in $\mathcal{G}$. The point where three or more rectangles of a given rectangular dual meet is called a joint. We know that a rectangular dual has $3-$joints and $4-$joints only where $4-$joints are regarded as a limiting case of $3-$joints [17]. Hence, abiding by the common design practice, we consider rectangular duals with $3-$joints in this paper. An interior face f of an oriented RDG is towards a vertex v if two edges of f with the same orientation are incident to v. Corresponding to a distinct regular edge labeling of an RDG, there is a topologically distinct rectangular dual of the RDG.

Theorem 1. [20, Theorem 2] An edge e or a block B formed by the four edges incident at an interior vertex of degree 4 of an oriented RDG is a changeable edge set if and only if one of the following is true:

i. the four boundary edges of e or of B have alternating orientations,

ii. e is a boundary edge and the interior face containing e is towards a corner vertex.

Definition 1. [12] A single edge in a rectangular dual $\mathcal{F}$ (or in its RDG) is a *turnable structure* (T−structure) if it occurs in any one of the four configurations shown in Fig. 3a−3d (or in Fig. 3e−3h). The red edges in Fig. 3 are T−structures. A T−structure may consist of more than one edge. A *simple T−structure* A in $\mathcal{F}$ is defined as a T−structure for which there exist 4 edges in $\mathcal{F}$ that do not belong to A, but share endpoints with A. Correspondingly in the RDG, a simple T−structure is a 4−cycle enclosing at least one vertex. For more clarification, refer to Fig. 4.

Theorem 2. [12, Theorem 6] A necessary and sufficient condition for a set A of edges in a rectangular dual $\mathcal{F}$ to be a *simple T−structure* is that the subgraph A^* consisting of edges that are dual to A in the oriented extended graph $\mathrm{E}(\mathcal{G})$ representing $\mathcal{F}$ is the subgraph contained in the interior of a 4−cycle C (cycle of length 4).

A 4−cycle in a plane graph is called a complex 4−cycle if it encloses at least one vertex. A 4−cycle is maximal if it is not contained in other 4−cycle. A rectangular dual is slicing if it is obtained by repeatedly cutting (slicing) a rectangle into component rectangles.

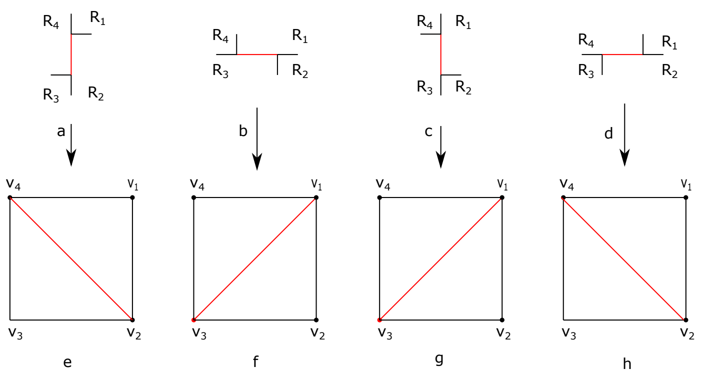

Figure 3. (a−d) Single edge simple T−structures in a rectangular dual and (e−h) Corresponding single edge simple T−structures in its RDG

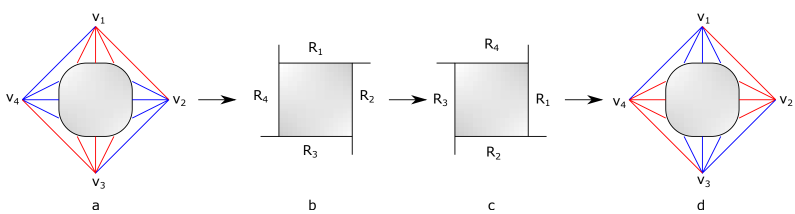

Figure 4. Multiple edge simple T−structures in $\mathcal{G}$

Theorem 3. [23, Theorem 1] If an RDG $\mathcal{G}$ contains no complex $4-$cycle, then it can be realized by a slicible rectangular dual.

Theorem 4. [5, Theorem 1] An RDG $\mathcal{G}$ with n vertices, $n > 4$, is slicible if it satisfies either of the following two conditions:

- its outermost cycle is the only complex $4-$cycle in $\mathcal{G}$ and at least one of its four vertices is a non$-$distinct corner;
- all the complex $4-$cycles of $\mathcal{G}$ are maximal.

Definition 2. [6] A rectangular dual is called *area$-$universal* if each assignment of areas to its rectangles can be realized by a combinatorially equivalent rectangular dual.

Definition 3. [6] A *line segment* in a rectangular dual is formed by a sequence of consecutive inner edges of the rectangular dual. A segment is *maximal* if it is not contained in any other segment.

Theorem 5. [6, Theorem 2] A rectangular dual $\mathcal{F}$ is *area$-$universal* if and only if every maximal internal line segment is the side of at least one rectangle of $\mathcal{F}$.

1.4. Results

Previous attempts [12, 13, 20] show that a number of topologically distinct rectangular duals can be realized from an RDG, i.e., regular edge labeling of a rectangular dual for a given RDG, in general, may not be unique. Yet, there exist a lot of RDGs in which each can be uniquely dualized (refer to Fig. 5) but the class of RDGs wherein each RDG can be uniquely dualized is still unknown.
In this paper, we derive a necessary and sufficient for a rectangularly dualizable graph to admit a unique rectangular dual upto combinatorial equivalence. Further we characterize the rectangularly dualizable graph by the fact that its *rectangular dual* is slicible and area$-$universal. Mathematically it is interesting to study this class since these RDGs has no alternative solutions, i.e., there is no need to recursively improve the solution.
In Section 2, we first present a necessary and sufficient condition for an RDG to admit a unique rectangular dual upto combinatorial equivalence. Then we show that this RDGs admits a slicible as well as an area$-$universal rectangular dual. Finally, we conclude our contribution in Section 3.

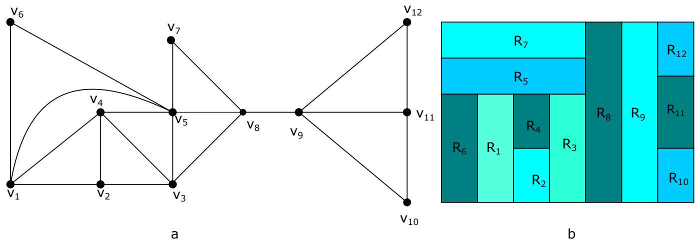

Figure 5. (a) An RDG that admits (b) a unique rectangular dual upto combinatorial equivalence.

2. Derivation of Unique Rectangularly Dualizable Graphs

In this paper, we derive a necessary and sufficient for a rectangularly dualizable graph to admit a unique rectangular dual upto combinatorial equivalence. Further we characterize the rectangularly dualizable graph by the fact that its *rectangular dual* is slicible as well as area−universal. For the sake of simplicity, we write a unique rectangular dual instead of a unique rectangular dual upto combinatorial equivalence throughout the section.

Theorem 6. Let $\mathcal{G}$ be an RDG having at least 4 vertices. A necessary condition for $\mathcal{G}$ to admit a unique rectangular dual is that $\mathcal{G}$ has exactly 4 vertices of degree 2.

Proof. Assume that $\mathcal{G}$ admits a unique rectangular dual $\mathcal{F}$. Denote the degree of a vertex v_k of $\mathcal{G}$ by $d(v_k)$. Let v_c be a vertex in $\mathcal{G}$ that corresponds to a corner rectangle R_c in $\mathcal{F}$. Since R_c is a corner rectangle in $\mathcal{F}$, its two adjacent sides are adjacent to the exterior. We claim that $d(v_c) = 2$. To the contrary, suppose that $d(v_c) > 2$. This implies that there exist rectangles R_1, R_2, ..., R_n $(n \geq 2)$ that are adjacent to the same side of R_c and one of them is an exterior rectangle. Denote it by R_e. Now the edges that have an endpoint incident to R_1 , R_2 , ..., R_n and the other endpoint incident to v_c have the same orientations (horizontal or vertical). Consequently, the inner face containing the boundary edge e_b joining v_c and the vertex dual to R_e is towards v_c. By Theorem 1, the orientation of e_b is changeable, which contradicts the fact that $\mathcal{G}$ admits a unique rectangular dual. This proves our claim. Similarly, the degree of vertices, that are duals to the remaining three corner rectangles of $\mathcal{F}$, can be shown to be equal to two. Hence the theorem. □

Consider the RDG $\mathcal{G}$, but not the E($\mathcal{G}$) shown in Fig. 2b. It admits more than one topological distinct rectangular duals as shown in Fig. 2a and 2c, although it has 4 vertices of degree 2; v_1, v_3, v_5, and v_7. Thus we see that the converse of Theorem 6 is not true.
It can be seen in Theorem 6 that the orientations of both boundary edges incident to a two degree vertex of an RDG is not changeable. This means that the orientations of edges of four corner rectangles in the corresponding rectangular dual are fixed and

hence the orientations of edges of all its exterior rectangles with the exterior are also fixed. Now we make the necessary condition in Theorem 6 more stronger so that it can be a sufficient condition. We first need to prove the following Lemma.

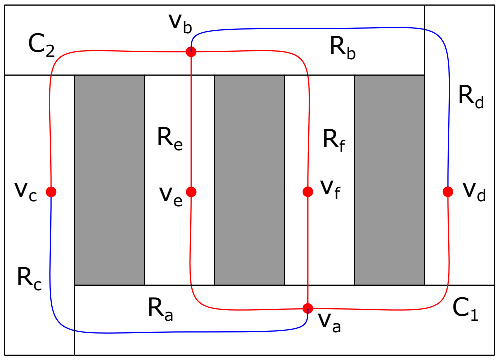

Figure 6. Two 4−cycles intersecting at two non−adjacent vertices of each other.

Lemma 1. Let $\mathcal{G}$ be an RDG. If C_1 and C_2 are 4−cycles in $\mathcal{G}$ intersecting each other at vertices v_a and v_b such that two edges of C_1 lie in the interior of C_2 and v_a, v_b are non−adjacent in both C_1 and C_2, then C_1 and C_2 never bound a T−structure.

Proof. The statement can be visualized using Fig. 6, where C_1 (passing through v_a, v_d, v_b, and v_e) intersects $\mathcal{C}_2$ (passing through v_a, v_c, v_b, and v_f) at two non−adjacent vertices v_a and v_b. Shaded areas contain other component rectangles. Note that the rectangles dual to a 4−cycle of an RDG enclosing some vertices, bound a rectangular area. Therefore the rectangles that are dual to the four vertices of $\mathcal{C}_2$ bounds a rectangular area enclosing the rectangle R_e which is dual to a vertex v_e of C_1. This implies that the edges incident to v_e and lying on C_2 have the same orientations which leads to a directed path consisting of only red edges (or blue edges) in the regular edge labeling of $\mathcal{G}$ joining v_a and v_b on C_1. This implies that C_1 is not a cycle of alternating edges in the regular edge labeling of $\mathcal{G}$ and hence by Theorem 1 or 2, C_1 never bounds a T−structure. Applying the same argument, we can show that C_2 never bounds a T−structure. Hence the result. □

Lemma 2. If $\mathcal{G}$ is an RDG, then each of its regions is triangular.

Proof. Assume that $\mathcal{F}$ is a rectangular dual that admits $\mathcal{G}$. No four rectangles of $\mathcal{F}$ meet at a point, i.e., $\mathcal{F}$ can have 3−joints only. Let $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ be any three interior rectangles of $\mathcal{F}$ which meet at a point or form a 3−joint. Then three vertices of $\mathcal{G}$ which are duals to these rectangles of $\mathcal{F}$ form a cycle of length 3 in the interior of $\mathcal{G}$. Hence each interior face (region) of $\mathcal{G}$ is triangular. □

Theorem 7. Let $\mathcal{G}$ be an RDG having at least 4 vertices. A necessary and sufficient condition for $\mathcal{G}$ to admit a unique rectangular dual is that $\mathcal{G}$ has 4 vertices of degree 2 and for any 4−cycle C_1 in $\mathcal{G}$, there is another 4−cycle C_2 intersecting C_1 at vertices v_a and v_b of $\mathcal{G}$ such that two edges of C_1 lie in the interior of C_2 where v_a and v_b are non−adjacent in both C_1 and C_2.

Proof. **Necessary Condition:** Assume that $\mathcal{G}$ admits a unique rectangular dual $\mathcal{F}$. By Theorem 6, it has four vertices of degree 2.
By Lemma 2, each regions of $\mathcal{G}$ is triangular. Then for $n > 3$, there always exists at least one 4−cycle in $\mathcal{G}$ consisting of a single edge or at least one vertex in its interior. To the contrary, assume that for a 4−cycle C_1 in $\mathcal{G}$, there is no another 4−cycle C_2 intersecting C_1 at vertices v_a and v_b of $\mathcal{G}$ such that two edges of C_1 lie in the interior of C_2 where v_a and v_b are non−adjacent in both C_1 and C_2. Then there are six possibilities for the occurrence of 4−cycles in $\mathcal{G}$:

i. a 4−cycle enclosing a single edge (see Fig. 7a),

ii. two 4−cycles C_p and C_q intersecting at two vertices v_a and v_b where v_a and v_b are non−adjacent in C_p and are adjacent in C_q (see Fig. 7d),

iii. a 4−cycle enclosing atleast one vertex (see Fig. 7g),

iv. two 4−cycles intersecting at vertex v_a such that one completely lies inside the other (see Fig. 7j),

v. two 4−cycles sharing an edge such that one completely lies inside the other (see Fig. 7k),

vi. two 4−cycles sharing two edges such that one completely lies inside the other (see Fig. 7l).

As shown in Fig. 7, the first three cases have T−structures. In Fig. 7j, the interior 4−cycle has no T−structure due to the well formedness of the outer 4−cycle. Since the outer 4−cycle encloses at least one vertex, it is similar to the 4−cycle shown in Fig. 7g and hence it has a T−structure.
If there exists another 4−cycle C containing this outer 4−cycle in its interior sharing a vertex, then C is similar to the 4−cycle shown in Fig. 7g and hence C has a T−structure and the interior two 4−cycles enclosed by it has no T−structure. If there is a chain of such 4−cycles with the property that the one which lies inside other shares a vertex, then the outermost 4−cycle in such chain has always a T−structure. Similarly, the outermost 4−cycle has a T−structure in Fig. 7k and 7l. Thus we have seen that all the six possibilities have T−structure which is a contradiction to the fact that $\mathcal{G}$ admits a unique rectangular dual $\mathcal{F}$. This proves the necessary part.

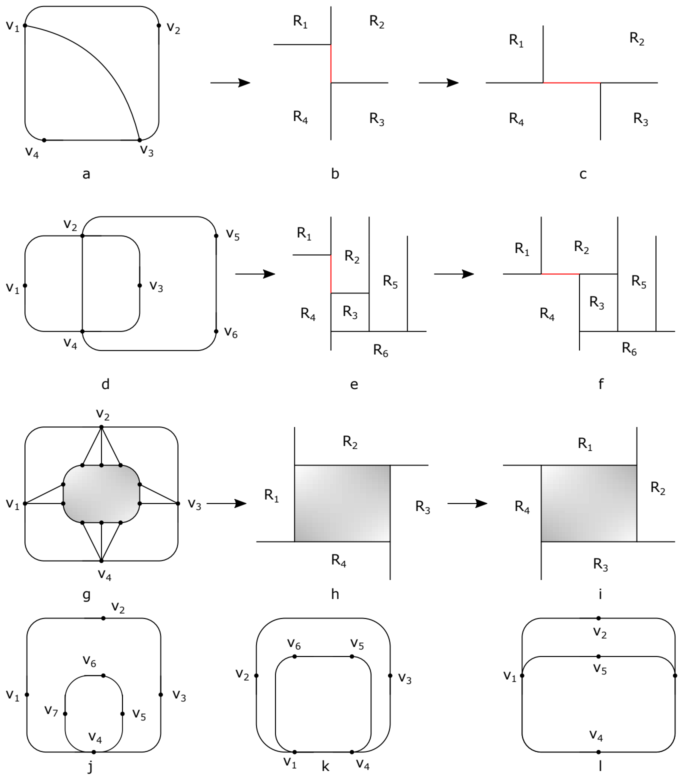

Figure 7. Demonstrations of all possible T$-$structures upto isomorphism.

Sufficient Condition: Assume that the given conditions hold. Note that none of the boundary edges incident to a vertex v_t of degree 2 can be towards v_t. By Theorem 1, the orientations of both boundary edges incident to v_t are not changeable. Also by Lemma 1, C_1 and C_2 never bound a T$-$structure. Consequently, $\mathcal{G}$ admits a unique rectangular dual. Hence the proof. □

Consider the RDG shown in Fig. 5. There is only one pair of 4$-$cycles satisfying Theorem 7: $v_1v_2v_3v_5v_1$ and $v_1v_4v_5v_6v_1$. Also the given RDG has four vertices v_6, v_7, v_{10}, and v_{12}, each of degree 2. Hence the given RDG admits a unique rectangular dual.

Theorem 8. If an RDG admits a unique rectangular dual $\mathcal{F}$, then $\mathcal{F}$ is slicible.

Proof. Assume that $\mathcal{G}$ is an RDG that admits a unique rectangular dual $\mathcal{F}$. If $\mathcal{G}$ has no 4$-$cycle, then by Theorem 3, $\mathcal{G}$ admits a slicible rectangular dual. If $\mathcal{G}$ has 4$-$cycles, then by Theorem 7 there only exist pairs of 4$-$cycles in $\mathcal{G}$ intersecting each other at vertices v_a and v_b of $\mathcal{G}$ such that the two edges of one of the two cycles (forming a pair) lie in the interior of the other where v_a and v_b are non$-$adjacent in both cycles. Clearly, none of them is contained in each other and hence both $4-$cycles are maximal. Since the pair of $4-$cycles is arbitrary, each $4-$cycle of $\mathcal{G}$ is maximal. Hence By Theorem 4, $\mathcal{G}$ admits a slicible rectangular dual $\mathcal{F}$. □

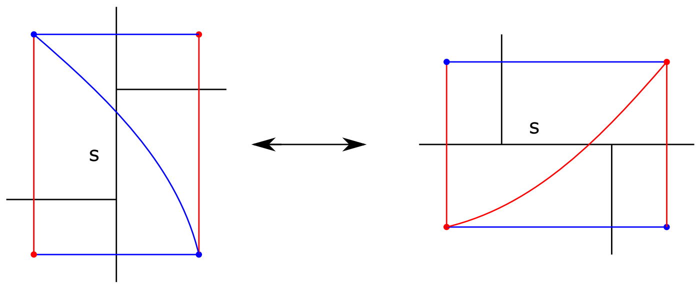

Figure 8. Multiple rectangles on both sides of a maximal line segment s.

Theorem 9. If an RDG admits a unique rectangular dual $\mathcal{F}$, then $\mathcal{F}$ is area$-$universal.

Proof. Assume that $\mathcal{G}$ is an RDG that admits a unique rectangular dual $\mathcal{F}$. To the contrary, suppose that $\mathcal{F}$ is not area$-$universal. By Theorem 5, there is a maximal internal line segment s in $\mathcal{F}$ which is not the side of any of its rectangles, i.e., s is a maximal internal line segment of $\mathcal{F}$ with multiple rectangles on both of its sides. Then an edge e of $\mathcal{F}$ (from which s is formed) must have one of its endpoints as a T$-$junction (a point where three rectangles meet) formed by the corners of two rectangles on one side of s, and on its other endpoint, it must have a T$-$junction formed by the corners of two rectangles on the other side of s, as shown in Fig. 8. Vertices dual to these four rectangles form a $4-$cycle of the alternating orientations in $\mathcal{G}$. By Theorem 1, this $4-$cycle is a changeable set which contradicts the fact that $\mathcal{G}$ admits a unique rectangular dual $\mathcal{F}$. Hence the result. □

3. Concluding Remarks

In general, the solution set (in terms of rectangular duals) of an RDG is very large. The crux of this study is to identify those RDGs whose solution set is singleton. In the literature, a series of papers studied transformations among rectangular duals of the oriented RDGs. Contrarily, we studied the uniqueness of rectangular duals by means of unoriented RDGs. Moreover, we characterized that each of these RDGs admits a slicible as well as an area$-$universal rectangular dual.

Conflict of interest. The authors declare that they have no conflict of interest.

Data Availability. Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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