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Research Article

Monophonic eccentric domination in graphs

P. Titus[1,*], J. Ajitha Fancy[2]

[1]Department of Mathematics, University College of Engineering Nagercoil, Anna University, Tirunelveli Region, Nagercoil - 629 004, India
titusvino@yahoo.com

[2]Department of Mathematics, Scott Christian College (Autonomous), Nagercoil - 629 003, India
ajithafancy@gmail.com

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Abstract: For any two vertices u and v in a connected graph G, the monophonic distance $d_m(u,v)$ from u to v is defined as the length of a longest $u-v$ monophonic path in G. The monophonic eccentricity $e_m(v)$ of a vertex v in G is the maximum monophonic distance from v to a vertex of G. A vertex v in G is a monophonic eccentric vertex of a vertex u in G if $e_m(u) = d_m(u,v)$. A set $S \subseteq V$ is a monophonic eccentric dominating *set* if every vertex in $V - S$ has a monophonic eccentric vertex in S. The monophonic eccentric domination number $\gamma_{me}(G)$ is the cardinality of a minimum monophonic eccentric dominating set of G. We investigate some properties of monophonic eccentric dominating sets. Also, we determine the bounds of monophonic eccentric domination number and find the same for some standard graphs.

Keywords: monophonic path, monophonic distance, monophonic eccentric vertex, monophonic eccentric dominating set, monophonic eccentric domination number

AMS Subject classification: 05C12, 05C69

1. Introduction

By a graph $G = (V,E)$ we mean a non-trivial finite undirected connected graph without loops and multiple edges. The *order* and *size* of G are denoted by p and q respectively. For basic graph theoretic terminology and results we refer to [1, 5]. For any two vertices u and v in G, the *distance* $d(u,v)$ is the length of a shortest $u-v$ path in G. The open neighborhood $N(v)$ of a vertex v is defined by $N(v) = \{u \in V \mid uv \in E\}$. A subset S of V is called a dominating set of G if $N(v) \cap S \neq \emptyset$

* *Corresponding Author*

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for all $v \in V - S$. A dominating set of G with minimum cardinality is a *minimum dominating set* and this cardinality is the *domination number* $\gamma(G)$. The topic of domination began with Berge in [1] and Ore in [13]. In 1998, a text book devoted to domination was written by Haynes et. al. [6]. A set $D \subset V(G)$ is an *eccentric dominating set* if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric vertex of v in D. The *eccentric domination number* $\gamma_{ed}(G)$ of a graph G equals the minimum cardinality of an eccentric dominating set. The eccentric domination number was introduced in [11] and further studied in [2, 3, 8–10, 12, 14]. For any two vertices u and v in G, the *detour distance* $D(u,v)$ is the length of a longest $u-v$ path in G. For each vertex v in G, define $D^-(v) = \min\{D(u,v) : u \in V - \{v\}\}$. A vertex u ($\neq v$) is called a *detour neighbor* of v if $D(u,v) = D^-(v)$. A vertex v is said to *detour dominate* a vertex u if $u = v$ or u is a detour neighbor of v. A set S of vertices of G is called a *detour dominating set* if every vertex of G is detour dominated by some vertex in S. A detour dominating set of G with minimum cardinality is a *minimum detour dominating set* and this cardinality is the *detour domination number* $\gamma_D(G)$. These concepts were introduced and studied in [4]. Also, detour eccentric domination number was introduced and studied in [7].

A *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called a *monophonic path* if it is a chordless path. For any two vertices u and v in a connected graph G, the *monophonic distance* $d_m(u,v)$ from u to v is defined as the length of a longest $u - v$ monophonic path in G. The *monophonic eccentricity* $e_m(v)$ of a vertex v in G is $e_m(v) = \max\{d_m(u,v) : u \in V\}$. A vertex v in G is a *monophonic eccentric vertex* of a vertex u in G if $e_m(u) = d_m(u,v)$. The monophonic distance was introduced in [15] and further studied in [16].

In this paper, we introduce the concept of monophonic eccentric domination and present a few basic results on the corresponding parameter. Also, we found one more variant of this new parameter called total monophonic eccentric domination number [18] and further studied in [17, 19].

2. Monophonic Eccentric Domination Number

Definition 1. Let v be any vertex of a connected graph G. The set of all monophonic eccentric vertices of v is called the *monophonic eccentric neighborhood* of v and it is denoted by $N_{e_m}(v)$. The *monophonic eccentric degree* of a vertex v is defined as $\deg_{e_m}(v) = |N_{e_m}(v)|$. The *minimum monophonic eccentric degree* $\delta_{e_m}(G)$ is defined as $\delta_{e_m}(G) = \min\{\deg_{e_m}(v) : v \in V\}$ and the *maximum monophonic eccentric degree* $\Delta_{e_m}(G)$ is defined as $\Delta_{e_m}(G) = \max\{\deg_{e_m}(v) : v \in V\}$.

Remark 1. In a graph G, if u is a monophonic eccentric vertex of v, then v need not be a monophonic eccentric vertex of u. Hence if u is an element of $N_{e_m}(v)$, then v need not be an element of $N_{e_m}(u)$. For example consider the path $P_3 := v_1v_2v_3$. It is clear that $v_1 \in N_{e_m}(v_2)$ and $v_2 \notin N_{e_m}(v_1)$.

Definition 2. A set $S \subseteq V$ in a graph G is a *monophonic eccentric dominating set* if every vertex in $V - S$ has a monophonic eccentric vertex in S. The *monophonic eccentric domination number* $\gamma_{me}(G)$ is the cardinality of a minimum monophonic eccentric dominating set of G.

Example 1. Consider the graph G given in Figure 1. For the vertices of the graph G given in Figure 1, the monophonic eccentric vertices are given in Table 1. From the Table 1, it is easily seen that no 1-element or 2-element subset of G is a monophonic eccentric dominating set of G and so $\gamma_{me}(G) \geq 3$. Now, the set $\{v_1, v_2, v_3\}$ is a monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 3$.

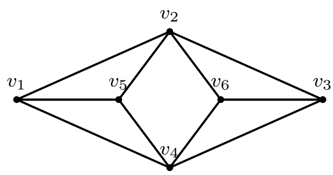

Figure 1. **A graph G with monophonic eccentric domination number 3**

Vertex	v_1	v_2	v_3	v_4	v_5	v_6
Monophonic eccentric vertices	v_3, v_6	v_4	v_1, v_5	v_2	v_3, v_6	v_1, v_5

Table 1. **Monophonic eccentric vertices of the graph illustrated in Figure 1**

Remark 2. For the path $P_n = v_1 v_2 \ldots v_n$, $n \geq 6$, $S = \{v_1, v_n\}$ is the unique minimum monophonic eccentric dominating set and hence it follows that the complement of a monophonic eccentric dominating set need not be a monophonic eccentric dominating set.

Theorem 1. *Let $G = K_{r,s}$ $(2 \leq r \leq s)$ be a complete bipartite graph. Then $\sum_{v \in V} \deg_{e_m}(v) = r^2 + s^2 - (r+s)$. Moreover, if S is any minimum monophonic eccentric dominating set of G, then $\sum_{v \in S} \deg_{e_m}(v) = r + s - 2$.*

Proof. Let $V_1 = \{u_1, u_2, \ldots, u_r\}$ and $V_2 = \{v_1, v_2, \ldots, v_s\}$ be the partite sets of G. For a vertex $u_i \in V_1$, $\deg_{e_m}(u_i) = r - 1$ and for a vertex $v_i \in V_2$, $\deg_{e_m}(v_i) = s - 1$. Then $\sum_{v \in V} \deg_{e_m}(v) = \sum_{v \in V_1} \deg_{e_m}(v) + \sum_{v \in V_2} \deg_{e_m}(v) = \sum_{v \in V_1}(r-1) + \sum_{v \in V_2}(s-1) = r(r-1) + s(s-1) = r^2 + s^2 - (r+s)$.
It is clear that $S = \{u_i, v_j\}$ $(1 \leq i \leq r, 1 \leq j \leq s)$ is a minimum monophonic eccentric dominating set of G. Hence $\sum_{v \in S} \deg_{e_m}(v) = \deg_{e_m}(u_i) + \deg_{e_m}(v_j) = r - 1 + s - 1 = r + s - 2$. □

Since no cut-vertex of a connected graph G is a monophonic eccentric vertex of any vertex in G, the following result is clear.

Remark 3. No cut-vertex of a connected graph G belongs to any minimum monophonic eccentric dominating set of G.

Theorem 2. *If $G = H + K_1$, where H is any connected graph, then $\gamma_{me}(G) = \gamma_{me}(H)$.*

Proof. Let u be the vertex of K_1. Since $d_{m_G}(u, z) = 1$ for any vertex z in H, every vertex of H is a monophonic eccentric vertex of u in G. Also, if $x, y \in V(H)$, then P is a longest $x-y$ monophonic path in H if and only if P is a longest $x-y$ monophonic path in G. Hence x is a monophonic eccentric vertex of y in H if and only if x is a monophonic eccentric vertex of y in G. Hence any minimum monophonic eccentric dominating set of H is also a minimum monophonic eccentric dominating set of G. It follows that $\gamma_{me}(G) \leq \gamma_{me}(H)$.
Now, let S_1 be any minimum monophonic eccentric dominating set of G. If $u \notin S_1$, then S_1 is also a monophonic eccentric dominating set of H. If $u \in S_1$, then $S_2 = (S_1 - \{u\}) \cup \{v\}$, where $v \in V(G)$ is a monophonic eccentric dominating set of H. Hence $\gamma_{me}(H) \leq \gamma_{me}(G)$. □

Next theorem gives the bounds of the monophonic eccentric domination number of a graph.

Theorem 3. *If k is the number of cut vertices of a connected graph G of order $p \geq 2$, then $1 \leq \gamma_{me}(G) \leq p - k$.*

Proof. Let T be the set of all cut vertices of G. It is clear that no cut vertex is a monophonic eccentric vertex of any vertex in G and every cut vertex has a monophonic eccentric vertex in G. Hence $S = V(G) - T$ is a monophonic eccentric dominating set of G and so $\gamma_{me}(G) \leq |S| = p - k$. The lower bound is obvious. □

Remark 4. The bounds in Theorem 3 are sharp. For the complete graph K_p $(p \geq 2)$, $\gamma_{me}(K_p) = 1$, and for the path P_p $(p \geq 4)$, $\gamma_{me}(P_p) = 2 = p - k$.

Theorem 4. *Let v be a vertex of a connected graph G of order $p \geq 2$ with $\deg_{e_m}(v) = \Delta_{e_m}(G)$. If v is a monophonic eccentric vertex of every vertex in $N_{e_m}(v)$, then $\gamma_{me}(G) \leq p - \Delta_{e_m}(G)$.*

Proof. Let $S = V(G) - N_{e_m}(v)$. Since v is a monophonic eccentric vertex of every vertex in $N_{e_m}(v)$, S is a monophonic eccentric dominating set of G and so $\gamma_{me}(G) \leq |S| = p - \Delta_{e_m}(G)$. □

Remark 5. The converse of Theorem 4 is false. For the graph G given in Figure 2, $\deg_{e_m}(v) = \Delta_{e_m}(G) = 3$ and $p = 4$. Also, $S = \{u\}$ is the unique minimum monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 1 = p - \Delta_{e_m}(G)$. But v is not a monophonic eccentric vertex of any vertex in $N_{e_m}(v) = \{u, x, y\}$.

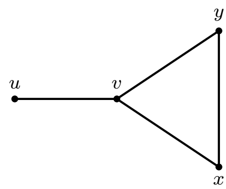

Figure 2. **The graph G given in Remark 5**

Based on Theorems 3 and 4 we leave the following problem as an open question.

Problem 1. Characterize graphs G of order $p \geq 2$ for which
(i) $\gamma_{me}(G) = 1$.
(ii) $\gamma_{me}(G) = p - k$, where k is the number of cut vertices of G.
(iii) $\gamma_{me}(G) = p - \Delta_{e_m}(G)$.

3. Monophonic Eccentric Domination Number of Some Standard Graphs

Theorem 5. *For the complete graph K_p $(p \geq 2), \gamma_{me}(K_p) = 1$.*

Proof. Since every vertex of the complete graph K_p $(p \geq 2)$ is a monophonic eccentric vertex of other vertices in K_p, any single vertex set is a minimum monophonic eccentric dominating set of K_p. Thus $\gamma_{me}(K_p) = 1$. □

Theorem 6. *For the path $G = P_n$, $\gamma_{me}(G) = \begin{cases} 1 & \text{if } n = 2, 3 \\ 2 & \text{if } n \geq 4. \end{cases}$*

Proof. Let $P_n := v_1 v_2 \ldots v_n$ be a path of order n. If $n = 2$ or 3, then $S_1 = \{v_1\}$ and $S_2 = \{v_n\}$ are the minimum monophonic eccentric dominating sets of G and so $\gamma_{me}(G) = 1$. If $n = 4$ or 5, then $S_1 = \{v_1, v_n\}, S_2 = \{v_1, v_2\}$ and $S_3 = \{v_{n-1}, v_n\}$ are the minimum monophonic eccentric dominating sets of G and so $\gamma_{me}(G) = 2$. If $n \geq 6$, then $S = \{v_1, v_n\}$ is the unique minimum monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 2$. □

Theorem 7. *For the star $G = K_{1,n}$, $\gamma_{me}(G) = 1$.*

Proof. Let $V_1 = \{u_1\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ be the partite sets of G. If $n = 1$, then $G = K_{1,1} = K_2$ and so by Theorem 5, $\gamma_{me}(G) = 1$. If $n \geq 2$, then $S = \{v\}$, where $v \in V_2$, is a minimum monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 1$. □

Theorem 8. *Let T be a tree with monophonic diameter at least 3. Then $\gamma_{me}(T) = 2$.*

Proof. Let T be a tree with monophonic diameter $d \geq 3$. Let x and y be any two monophonic diametral vertices of T such that $d_m(x, y) = d$. Claim that for any vertex u in T, $d_m(u, x) = e_m(u)$ or $d_m(u, y) = e_m(u)$. If $d_m(u, x) \neq e_m(u)$ and $d_m(u, y) \neq e_m(u)$, then there exists a monophonic eccentric vertex of u, say v, such that v is not an element of the $x - y$ monophonic path. Therefore, $d_m(u, v) = e_m(u) > \max \{d_m(u, x), d_m(u, y)\}$. When we consider x and y, assume that u lies nearer to x.

Case 1. u is a vertex of the $x - y$ monophonic path. Then

$$d_m(x, y) = d_m(x, u) + d_m(u, y) < d_m(x, u) + d_m(u, v) = d_m(x, v),$$

which is a contradiction.

Case 2. u is not a vertex of the $x - y$ monophonic path.
Since u and v do not lie on the $x - y$ monophonic path, let u_1 be the last common vertex of both $x - u$ and $x - y$ monophonic paths and let v_1 be the last common vertex of both $y - v$ and $y - x$ monophonic paths. It is clear that v_1 is the last common vertex of both $u - x$ and $u - v$ monophonic paths and u_1 is the last common vertex of both $v - y$ and $v - u$ monophonic paths. Therefore, $d_m(x, v_1) < d_m(v_1, v)$ and $d_m(y, u_1) < d_m(u_1, u)$. Now, $d_m(x, y) = d_m(x, v_1) + d_m(v_1, u_1) + d_m(u_1, y) < d_m(v_1, v) + d_m(v_1, u_1) + d_m(u_1, u) = d_m(u, v)$, which is a contradiction. Hence any vertex u in T is monophonic eccentric dominated by either x or y. Thus $S = \{x, y\}$ is a minimum monophonic eccentric dominating set of T and so $\gamma_{me}(T) = 2$. □

The next results follows from Theorem 7 and Theorem 8.
A *forest* is an acyclic graph in which each component is a tree.

Corollary 1. *If G is a forest containing k trees, then $\gamma_{me}(G) \leq 2k$.*

A *galaxy* is a forest in which each component is a star.

Corollary 2. *If G is a galaxy containing k components, then $\gamma_{me}(G) = k$.*

Theorem 9. *For the complete bipartite graph $G = K_{r,s} (2 \leq r \leq s)$, $\gamma_{me}(G) = 2$.*

Proof. Let $V_1 = \{u_1, u_2, \ldots, u_r\}$ and $V_2 = \{v_1, v_2, \ldots, v_s\}$ be the partite sets of G. It is clear that no single vertex set is a minimum monophonic eccentric dominating set of G. Then $S = \{u_i, v_j\} (1 \leq i \leq r, 1 \leq j \leq s)$ is a minimum monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 2$. □

Theorem 10. *If $G = K_1 + \cup m_j K_j$, then* $\gamma_{me}(G) = \begin{cases} 2 & \text{if } j \geq 2 \text{ and } \sum m_j \geq 2 \\ 1 & \text{otherwise.} \end{cases}$

Proof. Let $G = K_1 + \cup m_j K_j$ and let x be the vertex of K_1. We prove this theorem by considering three cases.

Case 1. $j \geq 2$ and $\sum m_j \geq 2$.
It is clear that x is not a monophonic eccentric vertex of any vertex in G. Since $\sum m_j \geq 2$, $G - x$ has at least two components. Let $u \neq x$ be a monophonic eccentric vertex of some vertex in G. Then u is a vertex of a component, say G_1, of $G - x$. Since $j \geq 2$, G_1 has at least one more vertex other than u, say v. It is clear that u is not a monophonic eccentric vertex of v. Hence a monophonic eccentric dominating set contains at least two vertices. Let $S = \{u, w\}$, where u and w belong to two different components, say G_1 and G_2, respectively. Then every vertex of $G - G_1$ is monophonic eccentric dominated by the vertex u and every vertex of $G - G_2$ is monophonic eccentric dominated by the vertex w. Hence S is a minimum monophonic eccentric dominating set of G and so $\gamma_{me}(G) = 2$.

Case 2. At least one $j = 1$ and $\sum m_j \geq 2$.
The graph G contains at least one end vertex, say u. It is clear that every vertex of $G - u$ is monophonic eccentric dominated by the vertex u and so $\gamma_{me}(G) = 1$.
Case 3. $j \geq 1$ and $\sum m_j = 1$.
The graph $G = K_1 + \cup m_j K_j$ is a complete graph. Then by Theorem 5, $\gamma_{me}(G) = 1$. □

Theorem 11. *Let $G = C_p$ $(p \geq 6)$ and let $p \equiv l \pmod 6$. Then*

$$\gamma_{me}(G) = \begin{cases} \lceil p/3 \rceil + 1 & \text{if } l = 2 \\ \lceil p/3 \rceil & \text{otherwise.} \end{cases}$$

Proof. Let $C_p : v_1, v_2, \ldots, v_p, v_1$ be a cycle having p vertices. Since every vertex in C_p has exactly two monophonic eccentric vertices, every vertex in C_p can monophonic eccentric dominates itself and at most two vertices in C_p, we have $\gamma_{me}(G) \geq p/3$. Let $p \equiv l \pmod 6$. We prove this theorem by considering six cases.

Case 1. $l = 0$.
It is clear that $S = \{v_1, v_2, v_7, v_8, \ldots, v_{p-5}, v_{p-4}\}$ is a monophonic eccentric dominating set of C_p. Since $\gamma_{me}(C_p) \geq p/3$, we have $\gamma_{me}(C_p) = p/3 = \lceil p/3 \rceil$.

Case 2. $l = 1$.
Let $S = \{v_1, v_4, v_7, v_{10}, \ldots, v_{p-3}, v_p\}$. It is easily verified that the vertices v_3 and v_{p-1} are monophonic eccentric dominated by v_1, the vertices v_2 and v_6 are monophonic eccentric dominated by v_4, the vertices v_5 and v_9 are monophonic eccentric dominated by $v_7, \ldots$, the vertices v_{p-5} and v_{p-1} are monophonic eccentric dominated by v_{p-3}, and the vertices v_2 and v_{p-2} are monophonic eccentric dominated by v_p. It is clear that S is a minimum monophonic eccentric dominating set of C_p and so $\gamma_{me}(C_p) = \lceil p/3 \rceil$.

Case 3. $l = 2$.
Let $S = \{v_1, v_4, v_7, v_{10}, \ldots, v_{p-4}, v_{p-1}\}$. It is easily verified that the vertices v_3 and v_{p-1} are monophonic eccentric dominated by v_1, the vertices v_2 and v_6 are monophonic

eccentric dominated by v_4, the vertices v_5 and v_9 are monophonic eccentric dominated by v_7,..., the vertices v_{p-6} and v_{p-2} are monophonic eccentric dominated by v_{p-4} and the vertices v_{p-3} and v_1 are monophonic eccentric dominated by v_{p-1}. But v_p is not monophonic eccentric dominated by any element in S. In a similar way it can be verified that no $\lceil p/3 \rceil$ element subset of V is a monophonic eccentric dominating set of C_p and hence $\gamma_{me}(C_p) > \lceil p/3 \rceil$. Let $S^{'} = S \cup \{v_p\}$. It is clear that $S^{'}$ is a monophonic eccentric dominating set of C_p and so $\gamma_{me}(C_p) = \lceil p/3 \rceil + 1$.

Case 4. $l = 3$.
Let $S = \{v_1, v_4, v_7, v_{10}, \ldots, v_{p-5}, v_{p-2}\}$. It is clear that S is a minimum monophonic eccentric dominating set of C_p and so $\gamma_{me}(C_p) = p/3 = \lceil p/3 \rceil$.

Case 5. $l = 4$.
Let $S = \{v_1, v_4, v_7, v_{10}, \ldots, v_{p-3}, v_p\}$. It is clear that S is a minimum monophonic eccentric dominating set of C_p and so $\gamma_{me}(C_p) = \lceil p/3 \rceil$.

Case 6. $l = 5$.
Let $S = \{v_1, v_2, v_7, v_8, v_{13}, v_{14}, \ldots, v_{p-4}, v_{p-3}\}$. It is clear that S is a minimum monophonic eccentric dominating set of C_p and so $\gamma_{me}(C_p) = \lceil p/3 \rceil$. □

Theorem 12. *Let* $G = W_p$ $(p \geq 7)$, *and let* $p \equiv l \pmod 6$. *Then*

$$\gamma_{me}(G) = \begin{cases} p/3 + 1 & \text{if } l = 3 \\ \lceil (p-1)/3 \rceil & \text{otherwise.} \end{cases}$$

Proof. Let $G = W_p = K_1 + C_{p-1}$ be the wheel with $V(K_1) = \{x\}$ and $V(C_{p-1}) = \{v_1, v_2, \ldots, v_{p-1}\}$. It is clear that x is not a monophonic eccentric vertex of any vertex in G, but any vertex in C_{p-1} is a monophonic eccentric vertex of x. Hence any monophonic eccentric dominating set of W_p is a monophonic eccentric dominating set of C_{p-1} and vice versa. Then by Theorem 11, we have

$$\gamma_{me}(W_p) = \begin{cases} \lceil (p-1)/3 \rceil + 1 & \text{if } l = 3 \\ \lceil (p-1)/3 \rceil & \text{otherwise.} \end{cases}$$

If $l = 3$, then p is a multiple of 3 and so $\lceil (p-1)/3 \rceil = p/3$ and the result follows. □

Conflict of Interest
The authors declare no conflict of interest in this paper.

Data Availability
Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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