Communications in Combinatorics and Optimization
Vol. 9, No. 3 (2024), pp. 485-495
https://doi.org/10.22049/cco.2023.28034.1422

CCO
Commun. Comb. Optim.

Research Article

New results on orthogonal component graphs of vector spaces over $\mathbb{Z}_p$

Vrinda Mary Mathew[†], Sudev Naduvath[*], and Thadathil Varghese Joseph[‡]

Department of Mathematics, CHRIST (Deemed to be University), Bangalore-560029, Karnataka, India
[†]vrinda.mathew@res.christuniversity.in
[*]sudev.nk@christuniversity.in
[‡]joseph.tv@christuniversity.in

Received: 8 October 2022; Accepted: 23 March 2023
Published Online: 28 March 2023

Abstract: A new concept known as the orthogonal component graph associated with a finite-dimensional vector space over a finite field has been recently added as another class of algebraic graphs. In these types of graphs, the vertices will be all the possible non-zero linear combinations of orthogonal basis vectors, and any two vertices will be adjacent if the corresponding vectors are orthogonal. In this paper, we discuss the various colorings and structural properties of orthogonal component graphs.

Keywords: Orthogonal component graph, coloring, planarity, unicyclic, centrality measures

AMS Subject classification: 05C25, 05C50

1. Introduction

Let V be an n-dimensional vector space over a field F. Then, any vector $v \in V$ can be expressed as the linear combination of the basis vectors in a unique manner. Recently, an extensive investigation has been carried out on the graphs derived from vector spaces and the properties and characteristics of different types of graphs constructed from finite-dimensional vector spaces (see [4–8, 13]).

One such type of graph is the non-zero component graphs of finite-dimensional vector space over the finite fields, which was introduced and explored in [4, 6]. The non-zero component graphs are those graphs whose vertices are all the possible non-zero linear

* *Corresponding Author*

combinations of the basis vectors in which any two vertices will be adjacent if the corresponding vectors share a common basis vector in their linear combination. A detailed study on the various properties, including coloring, domination, structural aspects and topological indices, has also been carried out in [9–12].
Further extending the work on graphs on vector spaces led to the study of a new type of graph wherein the vertex set remains the same as the non-zero component graph, and the adjacency was defined based on the orthogonality of the corresponding vectors in the vector space. This new graph, called orthogonal component graph of a finite-dimensional vector space, is defined as follows:

Definition 1. [12] Let V be an n-dimensional vector space over the field F with p elements. Let $B = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be an orthogonal basis of V. The *orthogonal component graph* of finite-dimensional vector space over the finite field, denoted by $G_F^{(V_n)}$, is the graph whose vertices are all the possible non-zero combinations of these orthogonal basis vectors and any two vertices u and v in the graph will be adjacent if $u.v = 0$.

A major observation was that this class of graphs was similar to the complement of the non-zero component graph when we consider certain fields and not in all cases, as it depended on the elements of the field under consideration as well as the orthogonality of the basis which we take to generate the elements of the vector space. Thus, it is still an open problem to generalise the complement of the non-zero component graph as, in general, it cannot be stated that they are complements of each other. But the same is true when the field under consideration is $\mathbb{Z}_p$ (see [12]). Thus, in this paper, we consider only the orthogonal component graphs of finite-dimensional vector space over the field $\mathbb{Z}_p$, which is also the complement of the non-zero component graph of the vector space over the field $F = \mathbb{Z}_p$.

Example 1. Consider the 2-dimensional vector space V_2 over $\mathbb{Z}_2$. Then the graph $G_{\mathbb{Z}_2}^{(V_2)}$ is shown in Figure 1.

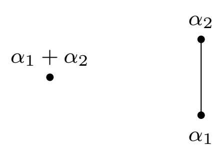

Figure 1. **The graph $G_{\mathbb{Z}_2}^{(V_2)}$**

Example 2. Consider the 3- dimensional vector space V_3 over $\mathbb{Z}_2$. Then the graph $G_{\mathbb{Z}_2}^{(V_3)}$ is shown in Figure 2.

Throughout our study, we consider the orthogonal component graph over an n-dimensional vector space over the field $\mathbb{Z}_p$. Hence, in this case, the adjacency of

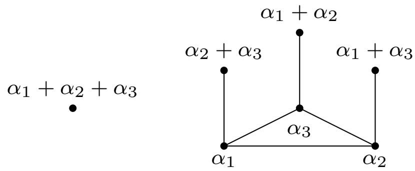

Figure 2. **The graph** $G_{\mathbb{Z}_2}^{(V_3)}$

any two vertices in $G_F^{(V_n)}$ can be redefined as, any two vertices u and v are adjacent if they do not share any common basis vectors in their representation as a linear combination of the basis. Also, the graph $G_F^{(V_n)}$ will always be a disconnected graph with p^n-1 vertices. In this paper we study some important properties of such graphs such as coloring, planarity, weak perfection and centrality measures. A graph G is said to be *weakly perfect* if $\chi(G)=\omega(G)$ for G, but not necessarily for all induced subgraphs of G. Centrality measures are those which helps in measuring the importance of various vertices in graphs.The various types of centrality are degree centrality, closeness centrality, betweeness centrality and eigen vector centrality. According to the degree centrality metric, a vertex's relevance in a graph is determined by its degree; the greater the degree of the vertex, the more significant it is in the graph. Since in the orthogonal component graph, the vertex which corresponds to the vector containing only one basis vector has got maximum degree, such vertices are more significant in the graph(See [12]). The closeness centrality measure of any vertex determines how near it is to every other vertex in the graph. Based on how frequently a vertex appears on the shortest path connecting all pairs of vertices in a graph, the betweeness centrality establishes and gauges the significance of a vertex in a network.

In this paper, we represent the orthogonal component graph of a finite-dimensional vector space $\mathbb{V}$ over a finite field $\mathbb{Z}_p$ by G instead of $G_{\mathbb{Z}_p}^{(V_n)}$. Also, throughout this discussion, by the term orthogonal component graph, we mean the orthogonal component graph of a given finite-dimensional vector space over the finite field $\mathbb{Z}_p$. Moreover, n represents $dim(V)$ and p represents $|Z_p|$.

For all basic terminologies and definitions in graph theory, algebraic graph theory and linear algebra, we refer to [1–3, 14, 15]

2. Coloring of orthogonal component graphs

In this section, we try to apply various graph colorings on an orthogonal component graph G over an n-dimensional vector space over the field F with p elements. Interestingly, it was found that the orthogonal graphs is one such type of graph class whose chromatic number, equitable chromatic number, Grundy chromatic number and co-chromatic number are the same.

The assignment of colors to the vertices of a graph such that no two vertices receive the same color is known as the *proper coloring* and the minimum number of colors used to obtain a proper coloring of the graph is known as the *chromatic number*. In this paper we also study various proper colorings of the orthogonal component graph G such as equitable coloring, Grundy coloring, complete coloring, cocoloring and obtain the respective chromatic numbers. The *equitable coloring* is another type of proper coloring in which the cardinalities of the color classes differ by at most 1. The minimum number of color classes formed in an equitable coloring is called the equitable chromatic number. In the following theorems, we calculate these chromatic numbers for an orthogonal component graph G.

Theorem 1. *The chromatic number of an orthogonal component graph G is equal to the dimension of V.*

Proof. Let G be an orthogonal component graph of an n-dimensional vector space V over the field $\mathbb{Z}_p$. Then, any vector in v can be expressed in terms of the n basis vectors, say $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. Let $\mathcal{S}(v)$ denotes the set of all basis vectors in v. The vertex set can be partitioned into n sets as follows, where $1 \leq k \leq n$.

$$V_k = \Big\{v \in V : |\mathcal{S}(v)| = k\Big\}.$$

Then, the cardinality of each of these sets will be

$$|V_k| = \binom{n}{k}(p-1)^k.$$

The basis vectors of V which are in V_1 are adjacent to each other and hence they need to be assigned different colors. The remaining vertices in V_1, which are the scalar multiples of the basis vectors can be assigned the same color as that of the corresponding basis vector as they are not adjacent in G. The vertices in V_2 are those which are expressed in terms of two basis vectors say, α_i and α_j. Therefore, the vertices containing these vectors can be assigned to those color classes containing α_i and α_j. In a similar way, all the vertices belonging to V_2 can be assigned to each color class. The vertices in V_3 are those consisting of three basis vectors. In a similar way, these vertices can be distributed into those color classes containing the respective basis vectors. Proceeding this way, all the vertices other than those expressed in terms of at most $n-1$ basis vectors will be assigned to any of those color class containing any of the respective missing basis vectors. It remains to assign colors to those vertices which are expressed in terms of all basis vectors. Since all such vertices are isolated, they can be assigned to any color class. Hence, the chromatic number is equal to the dimension of the vector space.

It remains to prove that the number of colors thus used to obtain a proper coloring of G is minimum. Since the clique number of the orthogonal component graph G is also $dim(V)$, we can conclude that the minimality is obtained. $\square$

Theorem 2. *The equitable chromatic number of the orthogonal component graph G is equal to the dimension of V.*

Proof. In order to prove the result, it suffices to show that $\chi_e(G) = \chi(G)$. The proper coloring of G mentioned in Theorem 2 also gives an equitable coloring of the orthogonal component graph G. The proper coloring is done in such a way that all those vertices containing k basis vectors are equally assigned to k color classes, which contains one of the basis vectors in the linear combination such that each of it receives equal number of vertices or at most one more than the other. The assignment is set up so that, at each iteration, the color class containing the least number of vertices gets the extra vertices if any when it is equally divided in the next step. Also, in each of these steps, the $\binom{n}{k}(p-1)^k$ number of vertices in the sets $V_k, 1 \leq k \leq n-1$, are equally distributed into n color classes. The remaining isolated vertices are also assigned to each of the color classes so that the difference between the cardinality of the color classes is either 0 or 1. Since the equitable chromatic number of G is equal to the chromatic number of G, the minimality is directly established. Hence, $\chi_e(G) = n$. □

Recall that the *Grundy coloring* is a proper coloring such that for every two colors i and j with $i < j$, every vertex colored j should have a neighbour colored i. The maximum number of colors that are used to obtain a Grundy coloring of G is called the *Grundy chromatic number*.

Theorem 3. *For an orthogonal component graph G, the Grundy chromatic number is equal to the dimension of the vector space V.*

Proof. In order to obtain a Grundy coloring, we partition the vertex set $V(G)$ into n sets in the following way such that each set receives the same color. Let W_1 be the set of vertices in G such that it contain all the vertices which contain the basis vector α_1 in the linear combination, W_2 be the set of vertices in G not belonging to W_1 which contains the basis vector α_2, W_3 be the set of vertices not belonging to W_1 and W_2 which contain the vector α_3. In a similar way, we get the sets $W_4, W_5, \ldots, W_n$. Since none of the vertices in each of the respective sets are adjacent to each other, each of these sets forms separate color classes. Also, this way of assigning colors gives a Grundy coloring of G, because all the vertices in W_2 will be adjacent to the vertex α_1 in W_1 as none of the vertices in W_2 contains α_1, thereby making all the vertices in W_2 which are assigned the same color adjacent to a vertex assigned a lesser color. The same follows for the other pair of colors.

It remains to prove that the Grundy number thus obtained is maximum.Since the W_i's partition the set of vertices of G into n independent sets, then a Grundy coloring gives the same color to all the elements in W_i, $1 \leq i \leq n$. Thus the Grundy number is at most n. Hence, the result follows. □

A *cocoloring* of the graph G is the assignment of colors in such a way that each color class forms an independent set in G or $\overline{G}$. The minimum number of colors used to obtain a cocoloring of G is called the cochromatic number of G.

Theorem 4. *The cochromatic number $z(G)$ of an orthogonal component graph G is equal to the dimension of vector space $dim(V)$.*

Proof. For an orthogonal component graph, the partitioning of the vertex set into subsets $W_i, 1 \leq i \leq n$ as discussed in Theorem 3 divides the vertex set into independent sets. Since each of these sets are independent, we can assign different colors to each of the sets, thereby producing a cocoloring of G.
Next, we need to prove that the number of colors thus obtained is minimum. Considering the orthogonal component graph G all the basis vectors need to be given different colors in G, and all the remaining vertices can be assigned to any of these color classes and on the other hand if we consider it's complement, the $(p-1)^n$ vertices itself form a clique and hence to be independent they need to be assigned to different color classes. Since the cochromatic number is the fewest of the number of colors, assigning colors to the independent sets in G itself would give the cochromatic number $z(G)$ which is equal to $dim(V)$. □

Corollary 1. *For an orthogonal component graph G, $\chi(G) = \chi_e(G) = \chi_{gr}(G) = z(G) = dim(V)$.*

Proof. The proof is immediate from Theorems 1,2,3 and 4. □

A *complete coloring* of G is a proper vertex coloring of G such that for every two distinct colors i and j used in the coloring, there exist adjacent vertices of G colored i and j. The maximum number of colors used to obtain a complete coloring of G is called the *achromatic number* of G. In the following theorem, we find the achromatic number of the orthogonal component graph G.

Theorem 5. *The achromatic number of an orthogonal component graph G over an n dimensional vector space over $\mathbb{Z}_p$ is:*

$$\chi_{ach}(G) = \begin{cases} dim(V) + 1, & \text{when } n > 2 \text{ and } p = 2 \\ dim(V), & \text{otherwise.} \end{cases}$$

Proof. Since by the coloring mentioned in Theorem 1, the basis vectors itself are assigned different colors and they form a clique, all pairs of colors exists and hence the corresponding coloring itself forms a complete coloring. But the achromatic number is the maximum number of colors used such that the resultant coloring gives a complete coloring. To prove that this coloring gives the achromatic number,we discuss the same by considering the following cases:

Case 1. When $n = 1$, since the graph is a complete disconnected graph, all the vertices are assigned the same color to obtain a complete coloring.

Case 2. When $n = 2$ and $q = 2$, the orthogonal component graph is isomorphic to $P_2 \cup K_1$, and in this case, maximum of two colors can be used so that there exists an edge corresponding to any pair of colors.

Case 3. When $n > 2$ and $p > 2$, we take any vertex other than these basis vectors and assign a different color, say l, to it other than those used. Since this vertex is not adjacent to those basis vectors which is present in the linear combination and assigned some color, say k, there will not exist a pair of colors k and p such that the corresponding vertices are adjacent. Hence the achromatic number of the orthogonal component graph will be $dim(V)$.

Case 4. When $n > 2$ and $p = 2$, even though the coloring mentioned in Theorem 1 gives a complete coloring, the achromatic number will not be equal to $dim(V)$ due to the presence of the pendant vertices in G. The pendant vertices are those vertices which will have only one basis vector missing in the linear combination. Hence if we assign a new color, say m, to those pendant vertices, then obviously there exists edges with end vertices having colors m and the already assigned n colors as each of these pendant vertex will be adjacent to the corresponding vertex which is expressed in terms of the missing basis vector and all the basis vectors are assigned n different colors. Hence in this case, achromatic number is n. The maximality of this follows in the same way as mentioned in Case 3. □

3. Some structural aspects of orthogonal component graphs

The orthogonal component graph G over a finite-dimensional vector space over the finite field $F = \mathbb{Z}_p$ is a tree or a disjoint union of trees when either V is 1-dimensional or 2-dimensional vector space over $\mathbb{Z}_2$. When $n = 1$, G will be a complete disconnected graph and hence it does not contain a cycle. Also, when $n = 2$ and $p = 2$, the resulting graph will not contain a cycle as the graph G will be $K_1 \cup P_2$. In all other cases, the graph G is not an acyclic graph. In particular, G will be unicyclic under the conditions mentioned in the following theorem.

Theorem 6. *The orthogonal component graph is unicyclic if and only if any of the following conditions holds.*

1. $dim(V) = 2$ *and* $F = \mathbb{Z}_3$;
2. $dim(V) = 3$ *and* $F = \mathbb{Z}_2$.

Proof. Let G be an orthogonal component graph of an n-dimensional vector space over a field F with p elements. If G is a graph which satisfies the given conditions, we will show that G is unicyclic.

Case 1. When $n = 2$ and $p = 3$, the set of vectors $\{\alpha_1, \alpha_2, 2\alpha_1, 2\alpha_2\}$ forms a C_4 and moreover it is the only cycle in G. Hence G is unicyclic.

Case 2. When $n = 3$ and $p = 2$, G will be $C_3 \circ K_1$, hence it is also unicyclic as it contains only one C_3.

Conversely, assume that G is not an orthogonal graph constructed with the conditions (1) or (2). Hence, in all the other cases, specifically when $n = 2$ and $p \geq 4$ or $n \geq 3$ and $p \geq 3$ or $n \geq 4$ and $p = 2$, there always exist at least two cycles $\{c_i\alpha_1, c_i\alpha_2, c_j\alpha_1, c_j\alpha_2\}$ and $\{c_i\alpha_1, c_i\alpha_2, c_k\alpha_1, c_k\alpha_2\}$ or $\{c_i\alpha_i, c_i\alpha_j, c_i\alpha_k\}$ and $\{c_j\alpha_i, c_j\alpha_j, c_j\alpha_k\}$ or $\{\alpha_i, \alpha_j, \alpha_k\}$ and $\{\alpha_j, \alpha_k, \alpha_l\}$ respectively. Hence, G is not unicyclic. This completes the proof. □

Theorem 7. *The orthogonal component graph is claw-free if and only if any of the following conditions holds.*

1. *$dim(V) = 1$;*
2. *$dim(V) = 2$ and $F = \mathbb{Z}_2$ or $\mathbb{Z}_3$;*
3. *$dim(V) = 3$ and $F = \mathbb{Z}_2$.*

Proof. Let us assume that G is an orthogonal component graph of an n-dimensional vector space over $\mathbb{Z}_p$. We first prove that when the given conditions (1), (2) or (3) holds, G will be claw-free.

Case 1. When $n = 1$, the orthogonal component graph will be a complete disconnected graph and hence it does not contain any $K_{1,3}$.

Case 2. When $n = 2$ and $p = 2$, $G = P_2 \cup K_1$, which is obviously claw free.

Case 3. When $n = 2$ and $p = 3$, $G = C_4 \cup 4K_1$, which is also a claw free graph.

Case 4. When $n = 3$ and $p = 2$, there will never exist a $K_{1,3}$ in G.

Conversely, we have to prove that if G do not satisfy either (1), (2) or (3), then G is not claw-free. Without considering those graphs which are constructed with the conditions in (1), (2) or (3), in all the other cases, specifically when $n \geq 2$ and $p \geq 4$ or $n = 3$ and $p = 3$, the orthogonal component graph G contains the set of vertices $\{c_i\alpha_i, c_i\alpha_j, c_j\alpha_j, c_k\alpha_j\}$ or $\{c_i\alpha_i, c_i\alpha_j, c_j\alpha_j, c_i\alpha_j + c_j\alpha_k\}$ which forms a claw in G. This proves the theorem. □

Proposition 1. *Every orthogonal component graph is weakly perfect.*

Proof. Since the chromatic number and the clique number of the orthogonal component graph G of a finite-dimensional vector space V over a field F is equal to $dim(V)$, and also when $dim(V) \geq 5$ there exists an induced C_5, we can say that G is weakly perfect. □

Theorem 8. *The orthogonal component graph is planar if and only $dim(V) \leq 4$ and $F = \mathbb{Z}_2$ or $\mathbb{Z}_3$.*

Proof. A graph G is planar if and only if G contains no subdivision of K_5 or $K_{3,3}$. First let us assume that G is an orthogonal component graph with $dim(V) \leq 4$ and $F = \mathbb{Z}_2$ or $\mathbb{Z}_3$.

Case 1. When $n = 1$, the graph is completely disconnected and hence planar.

Case 2. When $n = 2$ or $n = 3$ or $n = 4$ and $p = 2$, the orthogonal component graph G will be $P_2 \cup K_1$ or $C_3 \circ K_1$ or $G^{(\mathbb{Z}_2)}_{(V_4)}$ respectively, which are planar graphs as there does not exist a subdivision of K_5 or $K_{3,3}$ in the same.
Case 3. When $n = 2$ or $n = 3$ or $n = 4$ and $p = 3$, the corresponding graphs will be also be planar due to the absence of the subdivision of K_5 or $K_{3,3}$.

Conversely, let us assume that G is planar, we need to prove that G is an orthogonal graph over an $n-$dimensional vector space over the field F where $dim(V) \leq 4$ and $F = \mathbb{Z}_2$ or $\mathbb{Z}_3$. Let us assume the contrary. That is, assume that G does not satisfy the above condition. Then, we have to show that G is non-planar. In all the cases other than the condition mentioned above, specifically when $dim(V) \geq 5$ or $p \geq 4$, in an orthogonal component graph of an n-dimensional vector space over a field $\mathbb{Z}_p$, the presence of the set of vectors $\{\alpha_i, \alpha_j, \alpha_k, \alpha_l, \alpha_h\}$ or $\{c_i\alpha_i, c_j\alpha_i, c_k\alpha_i\}$ and $\{c_i\alpha_j, c_j\alpha_j, c_k\alpha_j\}$ induces a K_5 or $K_{3,3}$ graph respectively which in turn makes it non-planar. Hence the theorem. □

The closeness centrality of a vertex in an orthogonal component graph is discussed in the following theorem.

Theorem 9. *The closeness centrality of any vertex v whose corresponding vector contain k basis vectors in the connected component of the orthogonal component graph G of an n-dimensional vector space over the field with p elements is*

$$\mathbb{C}(v) = \begin{cases} \frac{p^n-(p-1)^n-1}{2p^n-p^{n-k}-2(p-1)^n-3}, & \textit{when } k < \frac{n}{2} \\ \frac{p^n-(p-1)^n-1}{2p^n-p^{n-k}+p^k-(p-1)^n-5}, & \textit{when } k \geq \frac{n}{2} \end{cases}$$

Proof. The closeness centrality is a measure of the average shortest distance from each vertex to each other vertex and is calculated using the formula

$$C(v) = \frac{|V(G)| - 1}{\sum\limits_{u \in V(G)} d(u, v)}.$$

For an orthogonal component graph, it can be established as explained below.
Let v be any vertex in G which does not contain any basis vectors in u. There are $p^{n-k} - 1$ such vectors and these will be adjacent to v. On the other hand, if u contains at least one basis vector contained in v, then u and v will be at a distance of 2 or 3. This can be considered int two different ways.

Case 1. If v contains less than half of the basis vectors of the vector space V(that is, $k < \frac{n}{2}$), then there are $p^n - 1 - (p^{n-k} - 1) - (p - 1)^n - 1$ number of vertices which are adjacent to v and having at least one basis vector common in the linear combination as that of u. Hence each of these $p^n - p^{n-k} - (p-1)^n - 1$ vertices will be at a distance 2 from v. Thus, altogether

$$\begin{aligned}C(v) = \frac{|V(G)| - 1}{\sum_u d(u,v)} &= \frac{p^n - (p-1)^n - 1}{2(p^n - p^{n-k} - (p-1)^n - 1) + p^{n-k} - 1}\\ &= \frac{p^n - (p-1)^n - 1}{2(p^n - (p-1)^n) - p^{n-k} - 3}.\end{aligned}$$

Case 2. If v contains more than half of the basis vectors of the vector space V(that is, $k \geq \frac{n}{2}$), then the vertices which do not have any common basis vector will be at a distance 1, those having common basis vectors missing in their corresponding linear combinations will be at a distance 2 and those do not have common basis vectors missing in their linear combinations will be at a distance 3. The vertices which are having common basis vectors missing in the linear combination are those which contains any of these $n-k$ vectors in the linear combination but not all the basis vectors. The number of such vertices will be equal to $(p^{n-k} - 1)(p^k - 1) - (p-1)^n$. The remaining all the vertices in the connected component of G excluding the vertex v itself will be at a distance 3 from v. Hence, there are $p^n - 1 - [p^{n-k} - 1 + (p^{n-k} - 1)(p^k - 1) - (p-1)^n + (p-1)^n + 1] = 3(p^n - p^{n-k} - (p^{n-k} - 1)(p^k - 1) - 1)$ such vertices having distance 2 from v. Altogether,

$$\begin{aligned}C(v) = \frac{|V(G)| - 1}{\sum_u d(u,v)} &= \frac{p^n - (p-1)^n - 1}{3(p^k - 2) + 2(p^{n-k} - 1)(p^k - 1) - (p-1)^n + p^{n-k} - 1}\\ &= \frac{p^n - (p-1)^n - 1}{2p^n - p^{n-k} + p^k - (p-1)^n - 5.}\end{aligned}$$

Hence the theorem. □

4. Conclusion

In this paper, we have discussed the various colorings of orthogonal component graphs of a vector space over the field $\mathbb{Z}_p$. It was interestingly found that the chromatic number, equitable chromatic number, Grundy number, achromatic number and cochromatic number are equal. Various other properties were also discussed. Studying the different types of perfectness and other centrality measures of these class of graphs can be considered as a future scope for the study.

Acknowledgements: The first author would like to acknowledge the critical and creative suggestions given by her fellow researchers Dr. Charles Dominic, Mr. Libin Chacko Samuel and Mr. Toby B Antony, which improved the content and presentation style of the manuscript.

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability Statement: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

References

[1] L.W. Beineke, R.J. Wilson, and P.J. Cameron, *Topics in Algebraic Graph Theory*, vol. 102, Cambridge University Press, 2004.

[2] J.A. Bondy and U.S.R. Murty, *Graph Theory*, Springer, New York, 2008.

[3] G. Chartrand, T.W. Haynes, M.A. Henning, and P. Zhang, *From Domination to Coloring: Stephen Hedetniemi's graph theory and beyond*, Springer Nature, 2019.

[4] A. Das, *Non-zero component graph of a finite dimensional vector space*, Comm. Algebra **44** (2016), no. 9, 3918–3926.

[5] ———, *Non-zero component union graph of a finite-dimensional vector space*, Linear Multilinear Algebra **65** (2017), no. 6, 1276–1287. https://doi.org/10.1080/03081087.2016.1234577.

[6] ———, *On non-zero component graph of vector spaces over finite fields*, J. Algebra Appl. **16** (2017), no. 1, Article ID: 1750007. https://doi.org/10.1142/S0219498817500074.

[7] ———, *On subspace inclusion graph of a vector space*, Linear Multilinear Algebra **66** (2018), no. 3, 554–564. https://doi.org/10.1080/03081087.2017.1306016.

[8] C.D. Godsil and M.W. Newman, *The automorphism group and fixing number of orthogonality graph over a vector space*, SIAM J. Discrete Math. **22** (2008), no. 2, 683–692.

[9] V.M. Mathew and S. Naduvath, *On non-zero component graphs of finite dimensional vector spaces*, Lecture Notes Netw. Syst. (2021).

[10] ———, *Coloring of non-zero component graphs*, Communicated (2022).

[11] V.M. Mathew, S. Naduvath, and I.N. Cangul, *Some vertex degree-based topological indices of non-zero component graphs*, Communicated (2022).

[12] V.M. Mathew, S. Naduvath, and T.V. Joseph, *On orthogonal component graphs of vector spaces over the field $\mathbb{Z}_p$*, Proyecciones J. Math. (to appear).

[13] S. Ou and Y. Tan, *The automorphism group and fixing number of orthogonality graph over a vector space*, J. Algebra Appl. **20** (2021), no. 12, Article ID: 2350013. https://doi.org/10.1142/S0219498823500135.

[14] G. Strang, *Introduction to Linear Algebra*, vol. 3, Wellesley-Cambridge Press Wellesley, MA, 1993.

[15] D.B. West, *Introduction to Graph Theory*, Prentice hall Upper Saddle River, 2001.