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Research Article

Optimal coverage of borders using unmanned aerial vehicles

Mohammad Etezadi[1,*], Siamak Ashrafi[2], Mostafa Ghasemi[1,†]

[1]Faculty of sciences, Imam Ali University, Tehran, Iran
*muhammad.etezadi@yahoo.com
†mostafaghasemi1366@yahoo.com

[2]Department of Mathematics, Maragheh Branch, Islamic Azad University Of Maragheh, Iran
siamak.ashrafi@iau-maragheh.ac.ir

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Abstract: Unmanned Aerial Vehicles (UAVs) play a very important role in military and civilian activities. In this paper, the aim is to cover the borders of Iran using UAVs. For this purpose, two zero-one programming models are presented. In the first model, our goal is to cover the borders of Iran at the minimum total time (the required time to prepare UAVs to start flying and the flight time of the UAVs). In this model, by minimizing the total time of UAVs for covering the borders, the costs appropriate to the flight of UAVs (such as the fuel costs of UAVs) are also reduced. In the second model, which is mostly used in emergencies and when a military attack occurs on the country's borders, the aim is to minimize the maximum required time to counter attacks and cover the entire country's borders. The efficiency of both models is shown by numerical examples.

Keywords: Unmanned Aerial Vehicle, Facility Location Problem, UAV Routing, Graph

AMS Subject classification: 90C27, 90C10, 05C70, 90B35

1. Introduction

Border security has become one of the most important issues for countries at the present time. Many countries are investigating new technologies to protect their borders from potential threats. It is critical that countries should foresee and take measures to address threats from neighboring countries. Currently, developed countries use state-of-the-art technologies to protect their borders, such as Unmanned

* *Corresponding Author*

Aerial Vehicles (UAVs), satellite-based surveillance systems, and sensors. UAVs are remotely piloted or self-piloted aircraft that can carry cameras, sensors, communications equipment or other payloads. The UAVs have been used in a reconnaissance and intelligence-gathering roles since the 1950s, and more challenging roles are envisioned to include including combat missions such as situation development, battle management, and battle damage assessment. The UAV becomes especially important when the geological structure of the region has areas with very steep cliffs (read as [www.global security]").

Events, wars, and various political disputes in neighboring countries highlight how important it is to provide security at a country's borders. Various geographical restrictions at the borders can cause difficulties in tracking and taking action. The motivation behind the work is to address the monitoring of movements along the land borders of Turkey by unmanned aircraft. The cities on the borders have been taken as demand points. Existing airports in Turkey are chosen as possible hubs. This problem is related to "location-routing" and "transportation–location" problems in the literature, which have been active areas of research since 1970 [3], [13] and [12]. Paper [14] focused on a single allocation p-hub median in unmanned aircraft system in order to monitor the movement at the land borders of Turkey.

In regard to the location-routing problem, there are many scholars doing the research after Jacobsen and Madsen [4] integrated the study of locations and routing problems in 1980. Tuzun et al. [17] promoted that LRP is an NP-hard problem. Min et al. [11] proposed a classification of location-routing problems based on facility capacities, vehicle capacities, and so on. Avellar et al. [1] present a solution for the problem of using a group of unmanned air vehicles (UAVs) equipped with image sensors to gather intelligence information, where the objective is to minimize the time to coverage of ground areas.

It is observed that UAVs are increasingly used for surveillance tasks. Since the maximum flight time of a UAV is limited, it is very important to have an optimal route plan to cover more points, lines or areas of interest (Sandar and Ratinam [15]). In addition, the authorized operation time may be more limited than the flight time.

Kress & Royst [6] proposed an optimization model to deploy and employ unmanned aerial vehicles (UAVs) in special operations missions.

Tian et al. [16] solved the problem of routing multiple drones to detect targets with a time window at a minimal cost. Constraints meet the segregation requirements for identifying targets and the requirement not to violate the maximum travel time. A GA-based approach is proposed to solve the problem. Liu et al. [7] consider the problem of drone routing as the task of monitoring traffic for a road network in which road sections are represented as nodes. The nodes are clustered by a clustering algorithm and then an iterative vendor problem is solved using a simulated annealing algorithm for each cluster. Liu et al. [9] and Liu et al. [8] introduced two multi-objective problems related to UAV route planning for collecting road traffic information. The complex integer linear programming formula and an exploratory approach to derive an optimal solution of the problem are proposed. Coelho et al. [2] report a routing problem for a heterogeneous UAV fleet that collects and delivers packets. Zhang et

al. [18] provide a linear integer programming formula to minimize detection delays and the cost of operations with limited flight paths in the field of traffic monitoring systems with fixed and movable sensors.
Karakaya and Sevinç [5] proposed a genetic algorithm as a solution approach to the problem of UAV routing to minimize the number of UAVs, and maximize the number of customers who provide services in the time window they need. Their aim is to determine the optimal location of air bases (airports) as well as the optimal allocation of border cities to established bases (each city to exactly one base) to control and monitor the borders of Turkey. For this purpose, the authors solved two general mathematical models. In the first model, the optimal location of the bases is determined first, and the second model is a routing model that optimally allocates cities to the established bases. The hypotheses and objectives of the paper [5] are as follows:

1: The speed of all UAVs is constant.

2: All UAVs are of the same type.

3: Each city is directly connected to an established base. More precisely, the border between a city and a base is actually an edge between the location of the base and the city (see Figure 1). This is due to the unique allocation hub location model used by the authors of this paper.

4: The number of bases that can be established is p. In other words, a limit is imposed on the maximum number of airports that can be established.

5: The objective function of the first model to determine the optimal location of the airports is maximum profit (minimum cost).

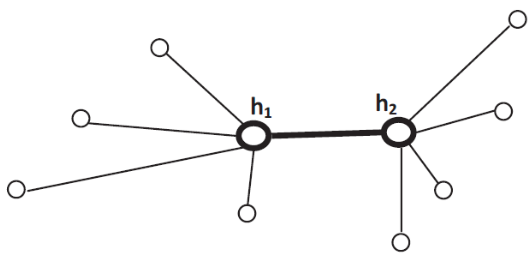

Figure 1. **Border network applied in the paper [5].**

The unmanned aerial vehicles also have broad applications in earthquake or tsunami relief and emergency communications. In [19], the authors by considering the uncovered area, the neighboring UAVs and the location of coverage boundary or obstacles, proposed a solution algorithm based on the simulation manner to cover the ground users by UAVs equipped with base station.

In [5], the authors have considered two types of costs: the cost of setting up a base and the cost of flying a UAV. Given that the cost of setting up a base is significant compared to the cost of flying, so minimizing the total establishment-costs and flight-costs do not seem to be an appropriate objective function. Further, the authors have assumed that all UAVs are of the same type. Also, the border between the two bases has not been considered, while between two established bases, there are definitely borders that need to be controlled and protected.

Unlike the previous studies, in our study,

- the trajectory of UAVs is a general graph
- in addition to the flight time, we also defined an initial preparation time, which is the time required for some initial checks on the UAVs to start flying from a base.
- the applied UAVs to control the borders have different types that differ in speed, initial preparation time and the maximum permissible time to fly continuously (according to the amount of fuel).

Compared to [5], in our first model, we define a different and more meaningful objective in which the goal is to minimize the total initial times required to prepare the UAVs to takeoff, and the flight time of the UAVs to control and monitor all available borders. Furthermore, in [5] it is not possible to pass form a city (as a node) twice. How UAVs move in [5] is shown in Figure (2). Considering the country's borders as a general graph, we will face with a situation that cities are not necessarily directly connected to the relevant base (See [14]), and to control the borders, a UAV can pass through several cities and eventually return to base. Our model will also have the ability to control the border connection between bases (unlike the paper [10]) and all stations and cities (as in the real world) will be connected (unlike Figure 2).

2. Problem statement and preliminaries

The use of UAVs to monitor, control and gather information from borders to prevent and counter military threats, especially for borders with geographical restrictions such as high cliffs and impassable routes is very necessary. It should be noted, however, that information obtained from border surveillance can also be used to manage the environment and transportation.

We know that in addition to the land borders, each country may also has some common maritime borders with neighboring countries, and both types of borders must be controlled. We consider the border network of Iran as a graph G. Therefore, assuming some vertices on the boundaries of the country and connecting them together, the borders of the country are determined more precisely. The edges between vertices

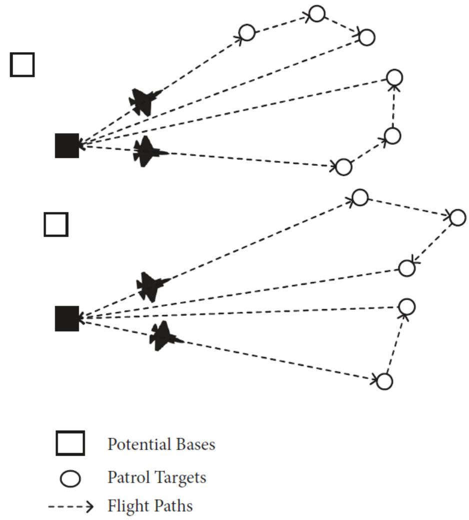

Figure 2. **Route motion of UAVs**

represent the movement paths of UAVs to control the borders. Due to the fuel limitation of UAVs, it may be necessary for a UAV to return to the base from a non-border air route after completely controlling some edges (borders between vertices). The non-border air routes are also considered as edges of graph G. Denote by $E_1(G)$ the set of all edges associated with the land and maritime borders. We called the edge $e \in E_1(G)$ a border-edge. Further, $E_2(G)$ indicates the edge set corresponding to the non-border air routes. Therefore, we have $E(G) = E_1(G) \bigcup E_2(G)$.

It is evident that a non-border air route (an edge in $E_2(G)$) is completely inside of the country. Therefore, it should be pointed out that the edges in $E_2(G)$ are only considered as air routes, and it is not necessary to be controlled by some UAVs. For example, in Figure 3, the edges $\{e_5, e_8, e_{11}, e_{13}, e_{18}, e_{21}, e_{22}\}$ represent common maritime borders with neighboring countries. Also, the edges $\{e_6, e_9, e_{12}, e_{19}\}$ are the borders separating land and sea, and these borders must be controlled to prevent illegal maritime transactions, military attacks by sea and etc. Further, the edges $\{e_3, e_7, e_{10}, e_{15}, e_{20}\}$, which are completely inside the country, are considered as the permissible air routes for the UAVs to pass. The remaining edges in Figure 3 are the common land-borders with neighboring countries.

Considering that each edge as the air route can be traversed in both directions by UAVs, we consider a directed graph to represent the borders of Iran. More precisely, $(\overrightarrow{G}, V(\overrightarrow{G}), E(\overrightarrow{G}))$ denotes the air routes as a general digraph where

$$V(\overrightarrow{G}) = V(G) \qquad , \qquad E(\overrightarrow{G}) = \{e = (v, w), \overleftarrow{e} = (w, v) |\ e \in E(G)\}.$$

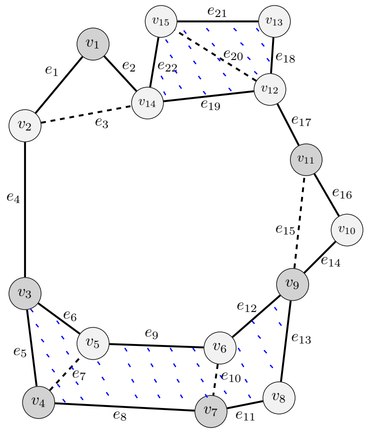

Figure 3. **A general graph as the borders of a country.**

Definition 1. We say that the edge $e = (v, w)$ is covered if and only if at least one UAV flies from v to w or from w to v.

A subset of vertex locations is considered as air bases. Unlike the previous studies, which assume that all UAVs are of a specific type, we consider a more general case in this study, and we assume that the UAVs used to cover the borders can be of different types that differ in terms of flight speed as well as the type of fuel consumed (nitrogen solution, gasoline or solid fuels). Due to the difference in the flight speed of the UAVs, the time required to travel a certain distance by different UAVs will be different. In addition, the time that a UAV can be in flight mode is limited and different for each type of UAVs. Every UAV needs some initial and necessary checks to start flying. We called the time spent for this operation as the preparation time. Note that the time required to prepare the UAVs may be spent to equip the reconnaissance UAVs with video and photography cameras or equipping the combat UAVs with guided bombs and lasers.
The UAVs are stationed at the candidate bases, and our goal is to select the suitable UAVs needed to fly and monitor the borders in such a way that all the borders of the country are covered. To represent the mathematical model of the problem, we first consider the following parameters:

$\overrightarrow{G}$: directed graph corresponding to the borders of the country.

$V(\overrightarrow{G})$: The vertices of $\overrightarrow{G}$.

$E(\overrightarrow{G})$: The edge set of $\overrightarrow{G}$.

I: The index set of vertices $V(G)$.

$S \subseteq I$: The index set of air bases.

$P_\ell[i,j]$: The l-th path between location v_i and location v_j on $\overrightarrow{G}$.

$\mathcal{P}_{j,i} = \{P_\ell[j,i] : \quad P_\ell[j,i] \subseteq \overrightarrow{G}\}$.

$\delta_i^+ = \{e = (v_i, w) : \quad e \in E(\overrightarrow{G})\}$.

$\delta_i^- = \{e = (w, v_i) : \quad e \in E(\overrightarrow{G})\}$.

$\mathcal{K}_j \subseteq K$: The index set of types of UAVs stationed in base v_j.

$\mathcal{R}_j^k$: The index set of UAVs of type k stationed at base v_j.

u_k^r: r-th UAV of type k.

U_j: Collection of all UAVs stationed at base v_j.

$t_e^{k,r}$: Time required to cover the edge e by the UAV u_k^r.

$t_0^{k,r}$: Time required to prepare UAV u_k^r for flight.

$\tau^{k,r}$: The maximum length of time that UAV u_k^r can flight with one refuel.

For example, $U_j = \{u_1^2, u_1^4, u_3^6\}$ means that the second and fourth UAVs of the first type and also the sixth UAVs of the third type are stationed at base v_j.
Furthermore, some fundamental assumptions are listed as follows:

- Each UAV lands in the same base it flew from (To prevent several UAVs to land on the same base, simultaneously, and also to maintain balance in the distribution of UAVs in different bases for future flights).

- Each edge $e \in E_1(G)$ must be covered.

- Without loss of generality, we assume that the required time to cover the edge $e = (i,j)$ by UAV of k-th type is the same as the required time to cover the edge $\overleftarrow{e} = (j,i)$, i.e., $t_e^{k,r} = t_e^{k,r'} = t_{\overleftarrow{e}}^{k,r'} = t_{\overleftarrow{e}}^{k,r}$.

- The required time for the initial preparation of the same type of UAVs is equal, i.e., $t_0^{k,r} = t_0^{k,r'}$ for all $r, r' \in R^k$.

- UAVs with the same type have the same speed.

Now, we define the decision variables $y_e^{k,r}$ and $x_j^{k,r}$ as

$$y_e^{k,r} = \begin{cases} 1, & \text{If border } e \text{ is covered by UAV } u_k^r, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

$$x_j^{k,r} = \begin{cases} 1, & \text{If UAV } u_k^r \text{ is selected to flight from base } v_j, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

In the continuation of this study, we present two zero-one programming models to cover all borders of Iran in calm and emergency cases and to demonstrate the efficiency of the proposed models, we solve the relatively large-scale numerical examples using optimization software (GAMS).

2.1. The Min-Sum model

As stated, our purpose is to cover, control and surveillance of the borders of Iran using UAVs. In the first model, we aim is to minimize the total required time for initial operations on UAVs, and required time for flight of UAVs to control all borders of the country. According to the above statements, the first model can be formulated as the following optimization model:

$$\mathrm{Pr}_1: \min \sum_{\substack{j\in S\\ k\in\mathcal{K}_j\\ r\in\mathcal{R}_j^k}} \sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r} + \sum_{\substack{j\in S\\ k\in\mathcal{K}_j\\ r\in\mathcal{R}_j^k}} t_0^{k,r}\, x_j^{k,r}$$

s.t.

$$\sum_{\substack{j\in S\\ k\in\mathcal{K}_j\\ r\in\mathcal{R}_j^k}} \left(y_e^{k,r} + y_{\overleftarrow{e}}^{k,r}\right) \geqslant 1 \qquad \forall e\in E_1(G), \tag{3}$$

$$y_e^{k,r} \leqslant x_j^{k,r} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j, r\in\mathcal{R}_j^k,\ \forall e=(j,i)\in E(\overrightarrow{G}), \tag{4}$$

$$y_e^{k,r} \leqslant \sum_{\ell=1}^{|\mathcal{P}_{j,i'}|} \left\lfloor \frac{\sum_{e'\in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r}}{\left|E\left(P_\ell[j,i']\right)\right|} \right\rfloor \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k,\ \forall e=(i',i)\in E(\overrightarrow{G}),\ i'\neq j, \tag{5}$$

$$\sum_{e\in\delta_i^+} y_e^{k,r} = \sum_{e\in\delta_i^-} y_e^{k,r} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k,\ \forall i\in I, \tag{6}$$

$$\sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r} \leqslant \tau^{k,r}\, x_j^{k,r} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k, \tag{7}$$

$$y_e^{k,r}\in\{0,1\} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k,\ \forall e\in E(\overrightarrow{G}), \tag{8}$$

$$x_j^{k,r}\in\{0,1\} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k. \tag{9}$$

In constraints (5), the notation $\lfloor \cdot \rfloor$ denotes a floor function. Note that $y_{e'}^{k,r} \in \{0,1\}$, then

$$\left\lfloor \frac{\sum\limits_{e' \in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r}}{|E\left(P_\ell[j,i']\right)|} \right\rfloor$$

is equal to zero or one in an optimal solution of model Pr_1. The set of constraints (3) states that all borders must be covered. As mentioned above, the edges in $E_2(G)$ are only the non-border air routes, and it is not necessary to cover them. The set of constraints (4) indicates that u_k^r can cover the edge $e = (j,i), j \in S$, if u_k^r flew from base v_j. Constraints (5) indicate the fact that u_k^r raised from base v_j can cover the border $e = (i',i), i' \neq j$, provided that it has previously covered all the edges on some paths $P_\ell[j,i']$. Precisely,

$$\sum_{e' \in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r} < |E\left(P_\ell[j,i']\right)|,$$

means that the path $P_\ell[j,i']$ is not covered by u_k^r, completey. In other words, there exists at least one edge $e' \in P_\ell[j,i']$ for which $y_{e'}^{k,r} = 0$. Now, if for all paths $P_\ell[j,i'] \in \mathcal{P}_{j,i'}$, we have

$$\sum_{e' \in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r} < |E\left(P_\ell[j,i']\right)|,$$

(there exist no path $P_\ell[j,i']$ so that completely covered by u_k^r) then it can immediately be concluded that

$$\sum_{\ell=1}^{|\mathcal{P}_{j,i'}|} \left\lfloor \frac{\sum\limits_{e' \in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r}}{|E\left(P_\ell[j,i']\right)|} \right\rfloor = 0.$$

In this case, the edge $e = (i',i), i' \neq j$, can not be covered by the UAV u_k^r, because $y_e^{k,r} \leqslant 0$. Moreover, the set of constraint (6) together with constraints (4) and (5) represent that if a UAV is selected for flight, it will land exactly at the same base which it flew from. The set of constraints (7) indicates that the total flight time of UAV u_k^r is less than the maximum permissible time for continuous flight of UAVs of type k. Finally, constraints (8) and (9) are the zero-one variables defined in equations (1) and (2).

It can be observed that model Pr_1 is not an integer programming model due to the nonlinear constraints (5). To obtain a 0-1 integer programming model equivalent to model Pr_1, we perform the following step:

Step 1. Define the new variables

$$\theta_{i',\ell}^{k,r} \in \{0,1\}, \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k, \qquad \forall i' \in I \setminus \{j\},\ \ \forall \ell = 1,...,|\mathcal{P}_{j,i'}|, \tag{10}$$

where

$$\theta_{i',\ell}^{k,r} \leqslant \frac{\sum\limits_{e' \in E\left(P_\ell[j,i']\right)} y_{e'}^{k,r}}{|E\left(P_\ell[j,i']\right)|}. \tag{11}$$

Step 2. Replace the set of constraints (5) by

$$y_{e'}^{k,r} \leqslant \sum_{\ell=1}^{|\mathcal{P}_{j,i'}|} \theta_{i',\ell}^{k,r}, \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k, \qquad \forall e = (i',i) \in E(\overrightarrow{G}),\ i' \neq j, \tag{12}$$

Step 3. Add the new variables (10), and the constraint (11) to model Pr_1.

Considering the objective function of Pr_1, by implementing the above steps, model Pr_1 converts to the following equivalent integer programming model:

$$\begin{aligned} \overline{\mathrm{Pr}_1}:\ \min \quad & \sum_{\substack{j\in S \\ k\in\mathcal{K}_j \\ r\in\mathcal{R}_j^k}} \sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r} + \sum_{\substack{j\in S \\ k\in\mathcal{K}_j \\ r\in\mathcal{R}_j^k}} t_0^{k,r}\, x_j^{k,r} \\ \text{s.t.} \quad & (3),\ \ (4),\ \ (12),\ \ (11),\ \ (6), \\ & (7),\ \ (8),\ \ (9),\ \ (10). \end{aligned}$$

Suppose that

$$\bar{x}_j^{k,r}, \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k,$$

$$\bar{y}_e^{k,r} \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \forall r \in \mathcal{R}_j^k,\ \forall e \in E(\overrightarrow{G}),$$

$$\bar{\theta}_{i',\ell}^{k,r} \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k, \qquad \forall i' \in I \setminus \{j\},\ \ \forall \ell = 1,...,|\mathcal{P}_{j,i'}|,$$

is an optimal solution of model $\overline{\mathrm{Pr}}_1$ with the optimal objective value $\bar{z}$. Then an optimal solution of the nonlinear programming model Pr_1 is obtained as

$$x^{*\,k,r}_j = \bar{x}^{k,r}_j, \qquad \forall j \in S,\ \forall k \in \mathcal{K}_j,\ \forall r \in \mathcal{R}^k_j \tag{13}$$

$$y^{*\,k,r}_e = \bar{y}^{k,r}_e \qquad \forall j \in S,\ \forall k \in \mathcal{K}_j,\ \forall r \in \mathcal{R}^k_j,\ \forall e \in E(\overrightarrow{G}), \tag{14}$$

with the optimal objective value $z^* = \bar{z}$.

2.1.1. Numerical example

Consider the graph displayed in Figure 3 as the borders of Iran, and the associated input data are presented in Table 1. Our goal is to control the borders to fight smugglers, monitor the oil pipelines, meteorological studies and etc, by existing UAVs stationed at bases so that the total flight time of UAVs and initial preparation times are minimized. We have three types of UAVs which are located at the bases $\{v_1, v_3, v_4, v_7, v_9, v_{11}\}$ where

UAVs stationed at base v_1: $\{u^1_1, u^1_3\}$

UAVs stationed at base v_3: $\{u^2_1, u^1_2\}$

UAVs stationed at base v_4: $\{u^3_1\}$

UAVs stationed at base v_7: $\{u^4_1, u^2_2\}$

UAVs stationed at base v_9: $\{u^3_2, u^2_3\}$

UAVs stationed at base v_{11}: $\{u^3_3\}$

- ▶ The speed of type 1 UAVs= 220 km/h,
- ▶ The speed of type 2 UAVs= 250 km/h,
- ▶ The speed of type 3 UAVs= 300 km/h.

Furthermore, the maximum permissible time to flight the UAVs, continuously, and the initial preparation time of UAVs are defined as

$$\tau^{1,r} = 9000 \text{ seconds} \quad , \quad t^{1,r}_0 = 1380 \text{ seconds} \quad \forall r \in \{1,2,3,4\},$$
$$\tau^{2,r} = 8400 \text{ seconds} \quad , \quad t^{2,r}_0 = 480 \text{ seconds} \quad \forall r \in \{1,2,3\},$$
$$\tau^{3,r} = 7200 \text{ seconds} \quad , \quad t^{3,r}_0 = 1200 \text{ seconds} \quad \forall r \in \{1,2,3\}.$$

Edge	Edge length (km)	Time required to cover the edge e by UAV of type 1 (seconds)	Time required to cover the edge e by UAV of type 2 (seconds)	Time required to cover the edge e by UAV of type 3 (seconds)
$e_1 = (1, 2)$	60	982	864	720
$e_2 = (1, 14)$	100	1636	1440	1200
$e_3 = (2, 14)$	90	1473	1296	1080
$e_4 = (2, 3)$	230	3764	3312	2760
$e_5 = (3, 4)$	130	2127	1872	1560
$e_6 = (3, 5)$	60	982	864	720
$e_7 = (4, 5)$	100	1636	1440	1200
$e_8 = (4, 7)$	200	3273	2880	2400
$e_9 = (5, 6)$	140	2291	2016	1680
$e_{10} = (6, 7)$	90	1473	1296	1080
$e_{11} = (7, 8)$	70	1145	1008	840
$e_{12} = (6, 9)$	100	1636	1440	1200
$e_{13} = (8, 9)$	150	2454	2160	1800
$e_{14} = (9, 10)$	100	1636	1440	1200
$e_{15} = (9, 11)$	130	2127	1872	1560
$e_{16} = (10, 11)$	110	1800	1584	1320
$e_{17} = (11, 12)$	80	1309	1152	960
$e_{18} = (12, 13)$	90	1473	1296	1080
$e_{19} = (12, 14)$	110	1800	1584	1320
$e_{20} = (12, 15)$	120	1964	1728	1440
$e_{21} = (13, 15)$	90	1473	1296	1080
$e_{22} = (14, 15)$	80	1309	1152	960

Table 1. **Input data associated with the Figure 3.**

Taking into account the above informations, the inpute data illustrated in Table 1, and solving model $\overline{\mathrm{Pr}}_1$ by GAMS software, and considering the relations (13), (14) an optimal solution of Pr_1 is obtained as:

$$x^{*3,1}_1 = 1, \quad x^{*2,1}_3 = 1, \quad x^{*1,3}_4 = 1, \quad x^{*2,2}_7 = 1, \quad x^{*2,3}_9 = 1, \quad x^{*3,2}_9 = 1,$$

$$y^{*2,1}_{e_1} = 1, \quad y^{*2,1}_{\overleftarrow{e}_1} = 1, \quad y^{*2,1}_{e_4} = 1, \quad y^{*2,1}_{\overleftarrow{e}_4} = 1,$$

$$y^{*3,1}_{e_2} = 1, \quad y^{*3,1}_{\overleftarrow{e}_2} = 1, \quad y^{*3,1}_{\overleftarrow{e}_{18}} = 1, \quad y^{*3,1}_{e_{19}} = 1, \quad y^{*3,1}_{\overleftarrow{e}_{21}} = 1, \quad y^{*3,1}_{e_{22}} = 1,$$

$$y^{*1,3}_{\overleftarrow{e}_5} = 1, \quad y^{*1,3}_{e_6} = 1, \quad y^{*1,3}_{\overleftarrow{e}_7} = 1,$$

$$y^{*2,2}_{e_7} = 1, \quad y^{*2,2}_{\overleftarrow{e}_8} = 1, \quad y^{*2,2}_{e_9} = 1, \quad y^{*2,2}_{e_{10}} = 1,$$

$$y^{*2,3}_{\overleftarrow{e}_{10}} = 1, \quad y^{*2,3}_{\overleftarrow{e}_{11}} = 1, \quad y^{*2,3}_{e_{12}} = 1, \quad y^{*2,3}_{\overleftarrow{e}_{13}} = 1,$$

$$y^{*3,2}_{\overleftarrow{e}_{14}} = 1, \quad y^{*3,2}_{e_{15}} = 1, \quad y^{*3,2}_{\overleftarrow{e}_{16}} = 1, \quad y^{*3,2}_{e_{17}} = 1, \quad y^{*3,2}_{\overleftarrow{e}_{17}} = 1,$$

where the remaining variables are equal to zero. The optimal objective value $z^* = 44693$ means that the selected UAVs cover all the borders illustrated in Figure 3 in $z^* = 44693$ seconds. The way UAVs move to cover the borders is shown in Figure 4. Notice that, $x_9^{*3,2} = 1$ means that the second UAV of type 3 stationed at the base v_9 has been selected for flight, and $y_{e_4}^{*2,1} = 1$ indicates that the border e_4 has been covered by the first UAV of type 2.

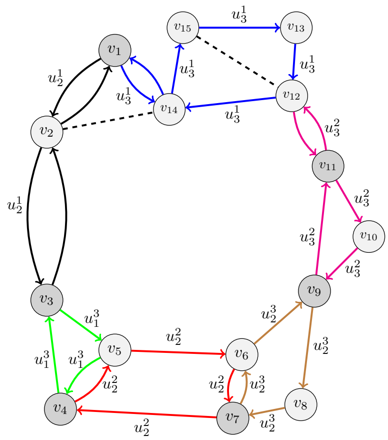

Figure 4. **How to move UAVs in Numerical example 2.1.1.**

2.2. The Min-Max model

Model Pr_1 minimizes the total flight time of the UAVs plus the time required to prepare the UAVs to fly and cover the borders of the country. In this model, which is mostly used in non-emergency situations to surveillance borders and collect information, we wish to minimize flight costs such as the cost of fuel consumed by UAVs, the cost of preparation by selecting the minimum number of UAVs needed to fliy, and etc, by minimizing the total flight time of UAVs. In the following, we examine another model in which the goal is to monitor and cover all the borders of the country in the shortest possible time. In the second model used in the emergency situations such as a military attack on the country, it is necessary to know the maximum time required to monitor and cover all the borders in order to take countermeasures to eliminate the threats.

For example, if three UAVs are selected to cover the entire borders and by considering the flight continuity limit, if the first UAV covers part of the country's borders in 5 minutes, the second UAV in 7 minutes and the third UAV in 10 minutes, then the maximum time to cover the entire borders is 10 minutes. This means that we need at least 10 minutes to be able to cover the entire borders according to the available UAVs to identify and destroy the threats.

According to the above description, in the model Pr_2, the goal is to minimize the maximum flight time of the UAVs, the time required to prepare the UAV to start flying plus the time the UAV is flying in the sky, according to the existing restrictions so that the entire borders of the country are covered. If u_k^r is selected to fly from the base where it is located, the time required to prepare the UAV u_k^r as well as the flight time of u_k^r to cover some borders will be calculated as follows:

$$\left(\sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r}\right) + t_0^{k,r}\, x_j^{k,r}.$$

Therefore, model Pr_2 can be formulated as the following optimization problem:

$$\mathrm{Pr}_2: \quad \min \max_{\substack{j\in S\\ k\in\mathcal{K}_j\\ r\in\mathcal{R}_j^k}} \left\{\left(\sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r}\right) + t_0^{k,r}\, x_j^{k,r}\right\}$$

s.t.

(3), (4), (5), (6),
(7), (8), (9).

The constraints of model Pr_2 are similar to the constraints of Pr_1.
Note that models Pr_1 and Pr_2 have completely different objective functions and have no relationship with each other. It is evident that in model Pr_2, applied in emergency situations, the UAVs should fly from several bases at the same time, and consequently, more UAVs are likely to be chosen to cover the borders.
It can easily be observed that model Pr_2 is a nonlinear optimization problem in which the objective function and constraints (5) are nonlinear. As in the previous section, to obtain a 0-1 integer programming model equivalent to Pr_2, we first replace the set of constraints (5) by the constraint (12). Furthermore, the constraints (11) and the new variables (10) are added to Pr_2. Model Pr_2 is still a nonlinear programming problem. Now by defining

$$z = \max_{\substack{j\in S\\ k\in\mathcal{K}_j\\ r\in\mathcal{R}_j^k}} \left\{\left(\sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r}\right) + t_0^{k,r}\, x_j^{k,r}\right\}$$

as a nonnegative variable, and adding the constraints

$$z \geqslant \left(\sum_{e\in E(\overrightarrow{G})} t_e^{k,r}\, y_e^{k,r}\right) + t_0^{k,r}\, x_j^{k,r} \qquad \forall j\in S,\ \forall k\in\mathcal{K}_j,\ \forall r\in\mathcal{R}_j^k \tag{15}$$

to the constrains of model Pr_2, we get

$$\begin{aligned}\overline{\mathrm{Pr}_2}: \quad \min \quad & z \\ \text{s.t.} \quad & (3),\ (4),\ (12),\ (11),\ (6), \\ & (7),\ (15),\ (8),\ (9),\ (10), \\ & z \geqslant 0.\end{aligned}$$

Observe that model $\overline{\mathrm{Pr}_2}$ is a 0-1 integer programming problem.
Now, if $\bar{z}$,

$$\bar{x}_j^{k,r}, \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k,$$

$$\bar{y}_e^{k,r} \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \forall r \in \mathcal{R}_j^k,\ \forall e \in E(\overrightarrow{G}),$$

$$\begin{aligned}\bar{\theta}_{i',\ell}^{k,r} \qquad & \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k, \\ & \forall i' \in I \setminus \{j\},\ \ \forall \ell = 1, ..., |\mathcal{P}_{j,i'}|,\end{aligned}$$

be a optimal solution of model $\overline{\mathrm{Pr}_2}$ with the optimal objective value $\bar{z}$, then an optimal solution of the nonlinear programming model Pr_2 is obtained as

$$x_j^{*k,r} = \bar{x}_j^{k,r}, \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \ \forall r \in \mathcal{R}_j^k, \tag{16}$$

$$y_e^{*k,r} = \bar{y}_e^{k,r} \qquad \forall j \in S,\ \ \forall k \in \mathcal{K}_j,\ \forall r \in \mathcal{R}_j^k,\ \forall e \in E(\overrightarrow{G}), \tag{17}$$

with the optimal objective value $z^* = \bar{z}$.

2.2.1. Numerical example

In the following, we describe a numerical example to illustrate our developed solution approach for the Pr_2 model on the border network of Figure 3. The location of bases, the UAVs stationed at each base, the speed of UAVs, and the values of $\tau^{k,r}$ and $t_0^{k,r}$ for all k and r are the same with the Numerical example 2.1.1. Further, the inpute data associated with Figure 3 are given in Table 1.
Our goal is to cover all the borders displayed in Figure 3, so that the maximum time to cover the borders is minimized. After solving model $\overline{\mathrm{Pr}_2}$ using the GAMS software

and considering the relations (16) and (17), an optimal solution of model Pr_2 consists of

$$
\begin{aligned}
& x_1^{*1,1}=1 \;,\; x_3^{*1,2}=1 \;,\; x_3^{*2,1}=1 \;,\; x_4^{*1,3}=1 \;, \\
& x_7^{*2,2}=1 \;,\; x_9^{*2,3}=1 \;,\; x_9^{*3,2}=1 \;,\; x_{11}^{*3,3}=1 \;, \\
& y_{e_1}^{*1,1}=1 \;,\; y_{\overleftarrow{e}_2}^{*1,1}=1 \;,\; y_{e_3}^{*1,1}=1 \;, \\
& y_{e_4}^{*2,1}=1 \;,\; y_{\overleftarrow{e}_4}^{*2,1}=1 \;, \\
& y_{e_5}^{*1,3}=1 \;,\; y_{\overleftarrow{e}_5}^{*1,3}=1 \;, \\
& y_{e_6}^{*1,2}=1 \;,\; y_{\overleftarrow{e}_6}^{*1,2}=1 \;,\; y_{e_9}^{*1,2}=1 \;,\; y_{\overleftarrow{e}_9}^{*1,2}=1 \;, \\
& y_{e_8}^{*2,2}=1 \;,\; y_{\overleftarrow{e}_8}^{*2,2}=1 \;, \\
& y_{\overleftarrow{e}_{10}}^{*2,3}=1 \;,\; y_{\overleftarrow{e}_{11}}^{*2,3}=1 \;,\; y_{e_{12}}^{*2,3}=1 \;,\; y_{\overleftarrow{e}_{13}}^{*2,3}=1 \;, \\
& y_{e_{14}}^{*3,2}=1 \;,\; y_{e_{16}}^{*3,2}=1 \;,\; y_{\overleftarrow{e}_{15}}^{*3,2}=1 \;, \\
& y_{e_{17}}^{*3,3}=1 \;,\; y_{\overleftarrow{e}_{17}}^{*3,3}=1 \;,\; y_{e_{18}}^{*3,3}=1 \;,\; y_{\overleftarrow{e}_{19}}^{*3,3}=1 \;,\; y_{e_{21}}^{*3,3}=1 \;,\; y_{\overleftarrow{e}_{22}}^{*3,3}=1 \;,
\end{aligned}
$$

where the other variables are equal to zero and the optimal objective value z^* is $z^* = 7926$. The value $z^* = 7926$ is associated with the UAV u_1^2 for covering the edges e_6 and e_9, and indicates that we need at least 7926 seconds to cover all the borders of the country. According to the optimal solution of model Pr_2, the movement of the selected UAVs to cover the entire border is shown in Figure 5.

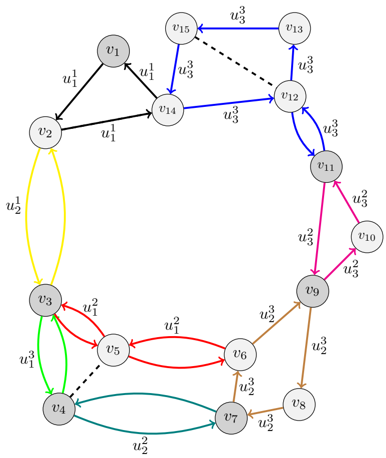

Figure 5. **How UAVs move to cover the borders in Numerical example 2.2.1.**

3. Conclusions

In this paper, the problem of covering the entire borders of Iran using UAVs was evaluated with two different objectives that are applied in emergency and non-emergency situations. In the first model, we developed an 0-1 integer programming model to minimize the total time required to control the entire borders of a country by UAVs. In the second model, which is applied in emergency situations, the maximum needed time to cover all the borders by existing UAVs was minimized by providing a 0-1 integer programming problem. Finally, numerical examples appropriate to both problems were presented to show the efficiency of the proposed zero-one programming models, and their optimal solutions were obtained using GAMS software in the shortest time. In addition, in the real world, the time required to travel a certain route by a UAV, which depends on the speed of the UAV, is not a fixed number due to weather conditions and has uncertainty. As well as the time required for initial preparation depends on the ability of the operator, technical faults in the UAVs, and other factors. Therefore, the times $t_0^{k,r}$ may also have uncertainty. Consequently, for future research, it would be interesting to investigate both models Pr_1 and Pr_2 in the uncertain space and present the heuristic or meta-heuristic algorithms to derive the optimal solution of these models.
Furthermore, due to the fact that the number of UAVs to control and protect of the borders has limitations, and on the other hand, some borders of the country due to the political events and military activities of the neighborhood near the border, are more important than other borders, it would be meaningful to present a new model to optimally allocate existing UAVs to higher priority and importance borders.

References

[1] G.S.C. Avellar, G.A.S. Pereira, L.C.A. Pimenta, and P. Iscold, *Multi-UAV routing for area coverage and remote sensing with minimum time*, Sensors **15** (2015), no. 11, 27783–27803.

[2] B.N. Coelho, V.N. Coelho, I.M. Coelho, L.S. Ochi, R. Haghnazar, D. Zuidema, M.S.F. Lima, and A.R. da Costa, *A multi-objective green UAV routing problem*, Compute. Oper. Res. **88** (2017), 306–315.

[3] L. Cooper, *The transportation-location problem*, Oper. Res. **20** (1972), no. 1, 94–108.

[4] S.K. Jacobsen and O.B.G. Madsen, *A comparative study of heuristics for a two-level routing-location problem*, Eur. J. Oper. Res. **5** (1980), no. 6, 378–387.

[5] M. Karakaya and E. Sevinç, *An efficient genetic algorithm for routing multiple UAVs under flight range and service time window constraints*, Int. J. Info.Technolog. **10** (2017), no. 1, Article ID 113.

[6] M. Kress and J.O. Royset, *Aerial search optimization model (ASOM) for UAVs in special operations*, Military Oper. Res. (2008), 23–33.

[7] X.F. Liu, L.M. Gao, Z.W. Guang, and Y.Q. Song, *A UAV allocation method for traffic surveillance in sparse road network*, Journal of Highway and Transportation Research and Development (English Edition) **7** (2013), no. 2, 81–87.

[8] X.F. Liu, Z.W. Guan, Y.Q. Song, and D.S. Chen, *An optimization model of UAV route planning for road segment surveillance*, Journal of Central South University **21** (2014), no. 6, 2501–2510.

[9] X.F. Liu, Z.R. Peng, Y.T Chang, and L.Y. Zhang, *Multi-objective evolutionary approach for UAV cruise route planning to collect traffic information*, Journal of central south university **19** (2012), no. 12, 3614–3621.

[10] Y. Liu, Z. Liu, J. Shi, G. Wu, and C. Chen, *Optimization of base location and patrol routes for unmanned aerial vehicles in border intelligence, surveillance, and reconnaissance*, J. Adv. Transport. **2019** (2019), Article ID 9063232.

[11] H. Min, V. Jayaraman, and R. Srivastava, *Combined location-routing problems: A synthesis and future research directions*, Eur. J. Ope. Res. **108** (1998), no. 1, 1–15.

[12] J.M. Nambiar, L.F. Gelders, and L.N. Van Wassenhove, *A large scale location-allocation problem in the natural rubber industry*, Eur. J. Oper. Res. **6** (1981), no. 2, 183–189.

[13] I. Or and W.P. Pierskalla, *A transportation location-allocation model for regional blood banking*, AIIE Trans. **11** (1979), no. 2, 86–95.

[14] İ. Sarıçiçek and Y. Akkuş, *Unmanned aerial vehicle hub-location and routing for monitoring geographic borders*, Appl. Math. Model. **39** (2015), no. 14, 3939–3953.

[15] K. Sundar and S. Rathinam, *Algorithms for routing an unmanned aerial vehicle in the presence of refueling depots*, IEEE Transactions on Automation Science and Engineering **11** (2013), no. 1, 287–294.

[16] J. Tian, L. Shen, and Y. Zheng, *Genetic algorithm based approach for multi-UAV cooperative reconnaissance mission planning problem*, Foundations of Intelligent Systems: 16th International Symposium, ISMIS 2006, Bari, Italy, September 27-29, 2006. Proceedings 16, Springer, 2006, pp. 101–110.

[17] D. Tuzun and L.I. Burke, *A two-phase tabu search approach to the location routing problem*, Eur. J. Oper. Res. **116** (1999), no. 1, 87–99.

[18] J. Zhang, L. Jia, S. Niu, F. Zhang, L. Tong, and X. Zhou, *A space-time network-based modeling framework for dynamic unmanned aerial vehicle routing in traffic incident monitoring applications*, Sensors **15** (2015), no. 6, 13874–13898.

[19] T. Zhao, H. Wang, and Q. Ma, *The coverage method of unmanned aerial vehicle mounted base station sensor network based on relative distance*, International Journal of Distributed Sensor Networks **16** (2020), no. 5, 1–12.