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Research Article

Linear-time construction of floor plans for plane triangulations

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Abstract: This paper focuses on a novel approach for producing a floor plan (FP), either a rectangular (RFP) or an orthogonal (OFP) based on the concept of orthogonal drawings, which satisfies the adjacency relations given by any bi-connected plane triangulation G.

Previous algorithms for constructing a FP are primarily restricted to the cases given below:

(i) A bi-connected plane triangulation without separating triangles (STs) and with at most 4 corner implying paths (CIPs), known as properly triangulated planar graph (PTPG).

(ii) A bi-connected plane triangulation with an exterior face of length 3 and no CIPs, known as maximal planar graph (MPG).

The FP obtained in the above two cases is a RFP or an OFP respectively. In this paper, we present the construction of a FP (RFP if exists, else an OFP), for a bi-connected plane triangulation G in linear-time.

Keywords: orthogonal floor plan, plane triangulation, orthogonal drawing, triconnected plane graph, algorithm

AMS Subject classification: 05C85, 68R10, 05C10, 05C62

1. Introduction

A floor plan F for a plane graph $G = (V, E)$ is a rectilinear polygon R which is partitioned into $|V|$ rectilinear polygons $P_1, P_2, \ldots, P_n$ called modules, where sides of every module are parallel to the sides of rectilinear polygon R. Every vertex $v_i \in V$ corresponds to a module P_i in such a way that if vertices v_1 and v_2 are adjacent in G,

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then the modules P_1 and P_2 share a common boundary in F. A FP with a rectangular boundary and composed of rectangular modules only is known as *rectangular floor plan* (RFP) whereas a FP with a rectangular boundary and composed of rectilinear modules is known as *orthogonal floor plan* (OFP). It is not always possible to construct a RFP for a plane triangulation G. In this case, it is necessary to construct an OFP for G. This problem is extensively studied in literature but restricted only to the class of bi-connected plane triangulations with the exterior face of length three, i.e., for maximal planar graphs (MPG [1]) [1, 3, 7, 10, 13, 17, 20, 32–35, 37, 38]. It has been proved in [38] that constructing an OFP for any bi-connected plane triangulation G must require 0, 1, and 2-CRMs [2]. In the proposed study, we propose an algorithm for constructing an OFP corresponding to any bi-connected plane triangulation G in linear-time with 0, 1, and 2-CRMs using the concept of orthogonal drawings.

1.1. Preliminaries

Here, we present a few important terminologies which are used frequently throughout this paper. For basic definitions related to graph theory which are not mentioned here, refer to [11].

A planar graph equipped with fixed embedding is said to be *plane graph* which divides the plane into number of connected components called as *regions/faces*. The unbounded/infinite region is called the *exterior face* (f_e) while all the remaining faces except f_e are called *interior faces*. A *contour* of a face is a cycle bounded by the edges on the boundary of a face. The boundary of f_e is a *contour* of G which is denoted by $C_o(G)$. A plane graph G is called *near triangulation* if its every interior face is a triangle; if its exterior face is also a triangle then G is called *triangulation*. A plane graph G is a *plane triangulation* if it is either a near-triangulation or a triangulation (see Figure 1(a)). A *separating triangle* (ST) in a plane triangulation G is a cycle of length 3 having at least 3 interior faces. A ST denoted as ST^c is *critical* if it has at least 4 interior faces. For example, in Figure 1(a), $\triangle v_1v_2v_3$ and $\triangle v_2v_4v_3$ are both STs whereas $\triangle v_1v_2v_3$ is a ST^c.

An *extended geometric dual* (G^*) is a type of geometric dual where a vertex corresponds to every interior face and every edge of the exterior face. There is an edge between two vertices if the corresponding faces of G shares a common edge or if vertices are lying on the exterior face (see Figure 2).

A simple cycle S of a plane graph G (i.e., a subgraph of G) with exactly k legs is *k-legged*, where a *leg* is an edge which does not belong to S and incident at exactly one vertex of S. For example, in Figure 3, C_2 is a 1-legged cycle (represented by purple cycle), where v_8v_7 is a leg. C_3 and C_6 (represented by red cycles) are 2-legged,

[1] *A maximal planar graph (MPG) is a planar graph whose every face is bounded by a triangle and adding an edge would destroy its property of planarity.*

[2] *A k-concave rectilinear module (k-CRM) is a connected module which is a union of k rectangular modules.*

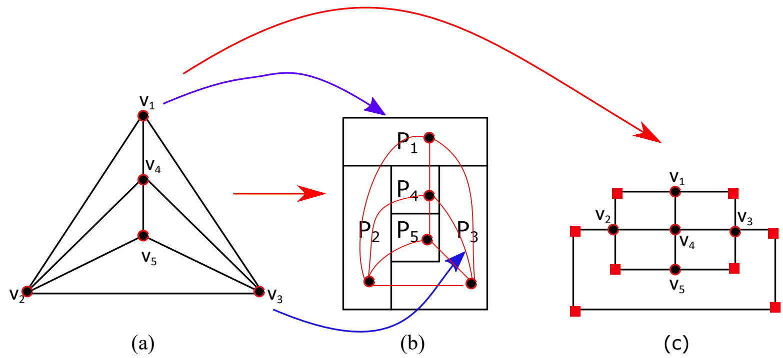

Figure 1. A plane triangulation G with its Orthogonal Floor plan and Orthogonal Drawing

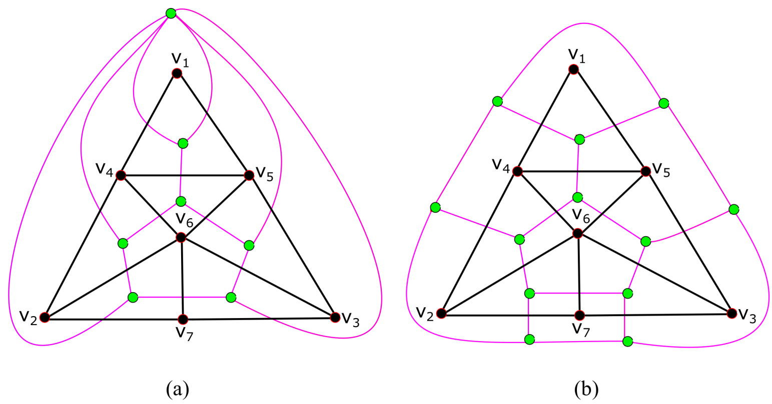

Figure 2. Illustration of the geometric dual and the extended geometric dual of a plane triangulation G. (a) The geometric dual G^+ (spanned by green vertices and pink edges) of G; (b) the extended geometric dual G^* (spanned by green vertices and pink edges) of G.

whereas C_1, C_4, and C_5 (represented by blue cycles) are 3-legged cycles. A simple cycle S which is either 1-legged, or 2-legged with at most one vertex of degree 2, or 3-legged cycle with no vertex of degree 2 is a *bad cycle*. For example, in Figure 3, all $C_i's$ $(1 \leq i \leq 6)$ are bad cycles. A bad cycle S is *minimal* if it contains no other bad cycle S' in itself. A bad cycle S is *maximal* if it contains other bad cycles and not contained in any other bad cycles. For example, in Figure 3, C_1, C_2, C_3 and C_5 are minimal bad cycles whereas C_6 is a maximal bad cycle. A *shortcut* is an edge in G whose vertices lies on $C_o(G)$ but not included in $C_o(G)$. A *corner implying path* (CIP) is a path $(w_1, w_2, w_3, \ldots, w_n)$ on $C_o(G)$, where (w_1, w_n) is a shortcut and $(w_3, w_4, \ldots, w_{n-1})$ is not a part of any other shortcuts. For example, in Figure 3, $v_{18}v_{20}$ is a shortcut, and $v_{18}v_{19}v_{20}$ is a CIP.

An *orthogonal floor plan* (OFP) for a plane triangulation G is a particular case of a FP where R is a rectangle. A concave corner in a module of an OFP is called

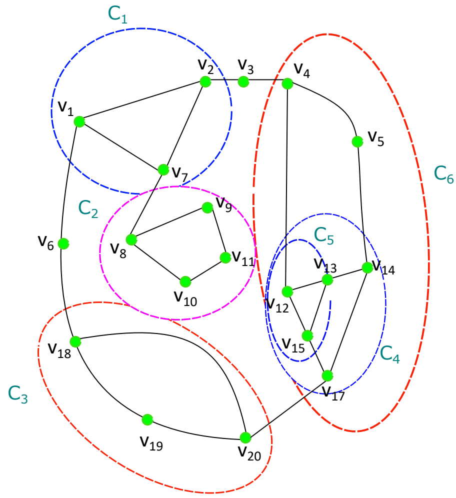

Figure 3. **Bad cycles, shortcut and corner implying paths**

bend. For example, Figure 1(b) represents an OFP with a bend in module P_3, where adjacency graph is spanned by the red edges. An *orthogonal drawing* (OD) is a plane embedding of a plane graph G where each edge is represented by consecutive sequence of horizontal and vertical line segments. The point of intersection where an edge changes its direction in an OD are called *bends* (see Figure 1(c), where red boxes represent bends). An OD which is either RFP or an OFP is called *orthogonal floor plan drawing* (OFPD). A *rectangular drawing* (RD) is a drawing of a plane graph G in which every vertex is represented by a point and edges connecting those points are drawn by horizontal or vertical line segments with no edge crossing. Every face of a RD is represented by a rectangle.

Notations:

FP: floor plan, RFP: rectangular FP, OFP: orthogonal FP, RD: rectangular drawing, OD: orthogonal drawing, OFPD: orthogonal floor plan drawing, EGDR: extended geometric dual representation, ST: separating triangle, ST^c: critical separating triangle, $k-$CRM: $k-$concave rectilinear module, f_e: exterior face, CIP: corner implying path, $d(v)$: degree of a vertex, MPG: maximal planar graph, PTPG: properly triangulated planar graph, G^+: geometric dual of G, G^*: extended geometric dual of G, $C_o(G)$: contour of G .

1.2. Literature Review

In literature, the relation of a *planar graph* and its geometrical representation has been well studied. In the proposed study, we consider a specific type of OD for a triconnected cubic graph which uses the concept of RD. The proposed work is relevant to RDs and ODs; hence, the significant contributions in this direction are briefed here.

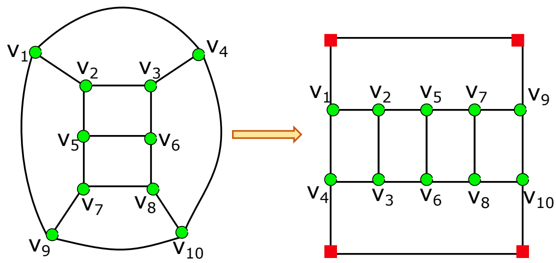

Figure 4. A triconnected cubic plane graph G and its RD with 4 designated corners which is a RFP as well

1.2.1 Contributions on Rectangular and Orthogonal Drawings

A drawing is a rectangular drawing (RD) if all of its faces can be drawn as rectangles. It is not always possible to draw a RD for a bi-connected plane graph having a vertex of maximum degree 3 [28]. Also, it cannot be always possible to draw a RD with four designated corners due to the presence of bad cycles. In literature, there are algorithms on the existence and construction of a RD of a plane graph [22, 26, 28]. They also provide work on the existence and construction of a RD with four designated corners for bi-connected plane graphs having a vertex of maximum degree 3 as well as for triconnected cubic planar graphs. According to [26, 28], RD with four designated corners is only possible if it contains no bad cycles (see Figure 4).

It is clear from the definition of an OD that an OD is possible only when every vertex has degree at most four (i.e., a plane triangulation with $d(v) \geq 5$ cannot have an OD). There are a lot of studies in the literature on ODs, some of the most suitable work in this direction are briefed here. In 1991, Tamassia *et al.* [36] worked for bi-connected planar graphs and gave the lower bound of $2n-2$ bends. They showed that a bend optimal drawing includes a single edge of length $\Omega(n^2)$ with $\Omega(n)$ bends along it. In 1994, Papakostas *et al.* [23] introduced an algorithm for a planar graph with vertex of maximum degree 4 which improved the bound on the number of bends. Even for a planar graph, the presented algorithm does not assure a planar drawing. Further, various approaches are used for ODs of bi-connected planar graphs, as mentioned in [6, 9]. In 2000, Nakano *et al.* [21] presented an algorithm for determining the bend-optimal OD in linear-time for bi-connected cubic planar graphs. In 2002, Rahman *et al.* [25] gave an algorithm for constructing an OD with the minimum number of bends corresponding to a triconnected cubic planar graph in linear-time. He also proved that an OD with minimum bends corresponding to a triconnected cubic planar graph is $b(G) + 4 - k$ where k is the number of corner 3-legged cycles, $b(G) = \sum_{i=1}^{l} bc(C_i)$, $bc(C_i)$ is a bend count corresponding to C_i and C_i is a maximal 3-legged cycle in G. In 2002, Rahman *et al.* [27] presented a linear-time algorithm for the construction of bend minimum OD for 3-plane graphs, i.e., a plane graph with a vertex of maximum degree 3. In 2006, Rahman *et al.* [29] provided a necessary and sufficient condition to check the existence of a no bend OD corresponding to a 3-plane graph. They also provided an algorithm for finding such drawings if it exists, in linear-time.

1.2.2 Contributions to Floor plans

There has been a lot of work done in the literature on floor planning problem using graph theoretic techniques. The problem of floor planning is to construct a FP which is an arrangement of modules corresponding to the adjacencies of a given graph. In the literature, the construction of FPs began with the construction of RFPs. Then the graphs for which RFP do not exist, researchers have presented the construction of OFPs. The details of most relevant work for FPs are briefed here. In early 1960s, initial approach for the generation of RFPs was first presented by Levin [19]. He proposed a method which represents the adjacencies of a FP in form of a graph. From 1970s to 1980s, most of the work has been done in the direction of existence and construction of RFPs for the class of PTPGs (PTPGs [3]) ([3, 5, 12, 14–16, 18, 30, 31]). In 1990s, researchers found that there does not exist RFPs for all planar triangulated graphs([32, 37, 38]) and proposed the construction of OFPs. In 1993, Sun and Sarrafzadeh [35] identified MPGs for which RFP do not exist and gave an algorithm for the construction of a FP with 0-CRM and 1-CRM, for those MPGs. He proposed an algorithm that checks if given graph G admits an OFP with 1-CRM or not in $O(n^{3/2})$ time and present its construction, if exists, in $O(n^2)$ time. In continuation of this work, in 1993, Yeap and Sarrafzadeh [38] showed that there are MPGs that do not admit an OFP, if only 0-CRM and 1-CRM are permitted. It has also been proved that 2-CRM is necessary and sufficient for the construction of OFPs for MPGs. In continuation to this work, in 1999, He [13] gave an algorithm for constructing an OFP corresponding to a MPG with 0-CRM, 1-CRM and 2-CRMs particularly Z-modules and T-modules in linear time. In 2003, Liao et al. [20] presented an algorithm for constructing an OFP with 0-CRM, 1-CRM and 2-CRMs in linear-time with more restriction on shapes of modules, i.e., Z-modules are not required. More literature on OFPs can be found in ([1, 7, 8, 17]).

Gaps in Existing Literature

By reviewing the existing work on FPs, it can be observed that the construction of FPs are restricted to the specific classes of planar graphs as given below:

(a) **RFPs**: Bi-connected plane triangulation with exterior face of any length with no STs and no more than 4 CIPs [4, 5, 15, 16, 18].

(b) **OFPs**: Bi-connected plane triangulation with exterior face of length 3 which may or may not contains STs and may or may not have CIPs [1, 2, 7, 10, 13, 17, 20, 32, 33, 35, 37, 38].

Hence, this study proposes an algorithm for constructing a FP (RFP/ OFP) for a general class of bi-connected plane triangulation (i.e., a bi-connected plane triangulation with exterior face of any length which may or may not contains STs and can

[3] *A properly triangulated planar graph (PTPG) is a connected planar graph where every interior face has length 3, every internal vertex has degree ≥ 4 and every non-face cycle has length ≥ 4.*

have any number of CIPs) in linear-time.
The remaining paper is organized in the following manner: The following section describes the orthogonal drawing for any triconnected cubic plane graph, which is an essential part of our approach which is followed by the extended geometrical representation of G. This section also presents an algorithm that computes a FP for G in linear-time. Furthermore, an example is provided to demonstrate the working of our algorithm, where the aesthetics of the obtained FP can be verified. Finally, section 4 summarises the contributions and discusses the future scope.

2. Obtaining a Floor plan for a Plane Triangulation

An OFP of a plane triangulation is an arrangement that depicts the relative location of polygons. Here, the task is to transform a plane triangulation G into a RFP if exists otherwise into an OFP which requires the following steps:

(i) Obtain an extended geometric dual representation (EGDR) of G (denoted by G^*).

(ii) Present a linear constructive algorithm for an orthogonal floor plan drawing (OFPD) of G^*.

The obtain OFPD is the required floor plan for G. Here, we propose the construction of EGDR and OFPD of a bi-connected plane triangulation G. The EGDR is a generalization of geometric dual for simple connected graphs given in [3]. The concept of OFPD is motivated from ODs with some specific properties for becoming an OFP.

2.1. Extended Geometric Dual Representation (EGDR)

Given any bi-connected plane triangulation G with n vertices and m edges, the following sets are defined:

(a) F_i denotes the set of all the interior faces of G;

(b) F_e denotes the exterior face of G.

and by the definition of a plane graph, the following condition holds:

(a) $F_i \cap F_e = \emptyset$;

(b) $F_i \cup F_e = m - n + 2$ (i.e., the total number of faces in G);

(c) $|F_e| = 1$.

The steps of construction of EGDR (G^*) of G is presented in Algorithm 1.

Algorithm 1: EGDR

Input: A bi-connected plane triangulation $G(V, E)$ with exterior face of any length

Output: Extended geometrical representation of G (G^*)

1 Vertex insertion: Place a vertex u_i inside face f_i, $\forall\ f_i \in F_i$; and a vertex w_j next to every edge of face F_e in face F_e.

2 Edge insertion:

(a) Place an edge e_p^* joining vertices u_k and u_l, $k \neq l$ while crossing the edge e_p which is common to the faces f_k and f_l, where f_k, $f_l \in F_i$;

(b) Place an edge e_q^* joining vertices u_k and w_j while crossing the edge e_q which is common to the faces f_k and f_j, where $f_k \in F_i$ and $f_j \in F_e$;

(c) Join vertices w_i and w_j, $i \neq j$ by an edge while not crossing any edges to form a cycle of length $|F_e|$ in F_e.

3 Exit

For a better understanding of Algorithm 1, refer to Figure 5.

Theorem 1. An extended geometric dual representation EGDR (G^*) of a bi-connected plane triangulation G is simple, 3-regular and triconnected graph which has n^* vertices and m^* edges, where $n^* = 2(m - |F_i|)$ and $m^* = 3(m - |F_i|)$.

Proof. EGDR (G^*) of a bi-connected plane triangulation G has the following properties:

(a) G^* is a simple graph: G is bi-connected and triangulated (i.e, G has no pendant vertex (i.e., vertex of degree 1) and no interior vertex of degree 2). Therefore, G^* is a simple graph.

(b) G^* is 3-regular: In the vertex insertion, a vertex is inserted corresponding to every face f_i, $\forall\ f_i \in F_{it}$ and for every edge of face f_j, where $f_j \in F_{et}$ in G^*. In the edge insertion step, two vertices which are inserted in the vertex insertion step are adjacent when the corresponding faces are adjacent or if they lies in the same face while not crossing an edge to form a cycle. Hence, the graph obtained in this process has every vertex of degree 3. This implies that G^* is 3-regular.

(c) G^* is triconnected: G^* is triconnected as G is simple, connected and 3-regular. For more details refer to [3].

(d) G^* has n^* vertices and m^* edges, where $n^* = 2(m - |F_i|)$ and $m^* = 3(m - |F_i|)$.

$$n^* = |F_i| + m(F_e) \tag{1}$$

$$m(F_e) = 2m - 3|F_i|. \tag{2}$$

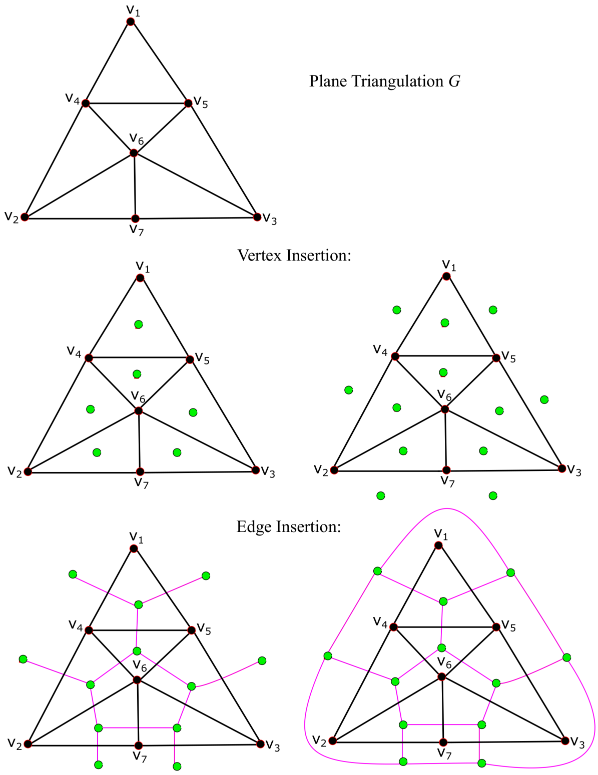

Figure 5. EGDR (G^*) of a plane triangulation G spanned by green vertices and pink edges

by Equations 1 and 2, It is clear that

$$\begin{aligned} n^* &= |F_i| + 2m - 3|F_i| \\ &= 2m - 2|F_i| \\ &= 2(m - |F_i|). \end{aligned} \tag{3}$$

According to Euler's formula for simple connected planar graph G

$$n - m + |F| = 2 \tag{4}$$

and according to Euler's formula for simple connected planar graph G^*

$$n^* - m^* + |F^*| = 2 \tag{5}$$

and

$$|F^*| = n + 1 \tag{6}$$

this implies that

$$n - 2 = m^* - n^* - 1 \tag{7}$$

by Equations 4 and 7

$$\begin{aligned} m^* &= m - |F| + n^* + 1 \\ &= m - |F_i| - |F_e| + n^* + 1. \end{aligned} \tag{8}$$

By Equations 3, 8 and $|F_e| = 1$ gives $m^* = 3(m - |F_i|)$.

□

2.2. Orthogonal floor plan drawing of G^*

In this part, we give an algorithm to generate a FP for a given plane triangulation G. To proceed, we first derive an OD for G^*. According to Lemma 1, the obtained G^* is a triconnected cubic graph and an algorithm is given by Rahman *et al.* [29] which constructs an OD for a triconnected cubic graph which is not an OFP. In this paper, we need to obtain an OD G^* which must be an OFPD. In the process of construction of an OFPD, we will use some ideas from the technique given by [29] for ODs.

Lemma 1 (Finding Bad Cycles and Rectangular Drawing construction [25]). Let G be a connected plane graph having four vertices of degree 2 on its contour and all other vertices are of degree 3 then G has a RD if and only if G has no bad cycles. One can find whether G has a bad cycle in $O(n)$ time and if it does, then a RD of G can be constructed in $O(n)$ time [28].

An illustration of Algorithm 2 is as follows: It's input is a bi-connected plane triangulation G having exterior face of any length, i.e., G can have STs or any number of CIPs. It's output is a FP which is either a RFP or an OFP. It first checks if there exist a RFP or an OFP for G then constructs corresponding EGDR (G^*) of G. It then calls Algorithm 3. Input for Algorithm 3 is G^* which is a triconnected cubic planar graph, i.e, G^* has no 1-legged or 2-legged cycles. G^* may contain a 3-legged cycle. For constructing OFPD of G^*, every 3-legged cycle of G^* must have at least four convex corners (inner angle of 90°). Hence, every 3-legged cycle corresponds to a corner in OFPD where a corner is a bend if it is not a vertex of G^*. We therefore have the facts for OFPD of G^* which are given as follows:

Fact 1. *In OFPD of G^*, exactly four bends must appear on its contour.*

Fact 2. *In OFPD of G^*, exactly one bend is required for every minimal 3-legged cycle.*

Algorithm 2: Generate FP

Input: A bi-connected plane triangulation G with exterior face of any length
Output: RFP or OFP

```
if G contains ST or # CIPs > 4 then
    Obtained FP is an OFP
else
    A RFP
    Call EGDR of G (Algorithm 1) // i.e.  G* which is triconnected cubic graph (see
    Section 3.1)
Call Modified OD (Algorithm 3)
Exit
```

Algorithm 3: Modified OD

Input: A Triconnected Cubic Graph G^*
Output: An Orthogonal Floor Plan Drawing
Data: Minimal Bad Cycles $C_i : 1 \leq i \leq l$ of G^* (for the computation of minimal bad cycles, refer to [24])

```
Let K be the set of all minimal bad cycles C_i : 1 ≤ i ≤ l
Let L ⊆ K, where L be the set of minimal bad cycles which lies on C_o(G*) and M = K − L,
where M be the set of remaining bad cycles which are not a part of C_o(G*)
if |L| ≤ 4 then
    Add a dummy vertex d_i on the edge common to C_i and C_o(G*), ∀ C_i ∈ L
    Add a dummy vertex h_j on any edge of C_j which is not a part of C_o(G*), ∀ C_j ∈ M
else if |L| > 4 then
    Add a dummy vertex d_i on the edge common to C_i, C_o(G*) and maximal bad cycle (if any),
    for any four C_i ∈ L
    Add a dummy vertex h_j on an edge of C_j which is not a part C_o(G*), ∀ C_j ∈ M and for
    remaining C'_j s ∈ L
Resulting graph is G*_1
Denote the number of vertices of degree 2 on C_o(G*_1) as b
if b ≤ 4 in G*_1 and G*_1 has outer face of length ≥ 4 then
    Insert 4 − b dummy vertices (p'_i s) on distinct edges of C_o(G*_1) having no endpoints of degree 2
else
    Add two dummy vertices (p'_i s) on an edge of C_o(G*_1) having no endpoints of degree 2 and a
    dummy vertex (p_j) on the remaining edges of C_o(G*_1) having no endpoints of degree 2
Resulting graph is G*_2
if |h'_j s| = 0 then
    Draw RD of G*_2 using [28]
    Obtained drawing is an OFPD of G which is a RFP
else
    Call a maximal bad cycle having h_i as T_i. Contract every T_i with a single vertex l_i and the
    resultant graph is G*_3
    Draw RD of G*_3 and of all T'_i s    Replace every l_i with its corresponding RD of T_i in the RD
    of G*_3
    Obtained drawing is desired OFPD of G which is an OFP
Exit
```

An outline of proposed Algorithm 3 is described as follows: Let G_1^* be a graph which is obtained from G^* by inserting dummy vertices $d_i's$ on the edges of minimal bad cycles $C_i's$ of G^* in the following manner: Insert a dummy vertex d_i on the edge

common to C_i and $C_o(G^*)$, for any four $C_i's$ lying on the outer boundary (see Figure 6). To fix the outer boundary of OFPD based on Fact 1, insert dummy vertices ($p_i's$) on the edges of $C_o(G_1^*)$ with no endpoint as d_i in such a way that $|p_i's| + |d_i's| = 4$ on $C_o(G^*)$ (see Figure 6). Insert dummy vertex h_i on any edge of C_i which is not a part of $C_o(G^*)$ based on Fact 2 (see Figure 8). In this way, the resulting graph G_2^* has no bad cycles.
By Lemma 1, if $|h_i's| = 0$ in G_2^* then G_2^* has a RD which is an OFPD, where the dummy vertices of $C_o(G_2^*)$ become the corners of rectangle which are bends in OFPD. If $|h_i's| \neq 0$ then contract every maximal bad cycle having a dummy vertex h_i with a single vertex l_i. The resulting graph G_3^* has only four dummy vertices which are a part of $C_o(G^*)$. Hence, we construct a RD of G_3^* and for each contracted bad cycle. Finally, we replace every l_i in the RD of G_3^* with the RD of the corresponding contracted bad cycles to get an OFPD of G^* (see Figures 8 and 10).

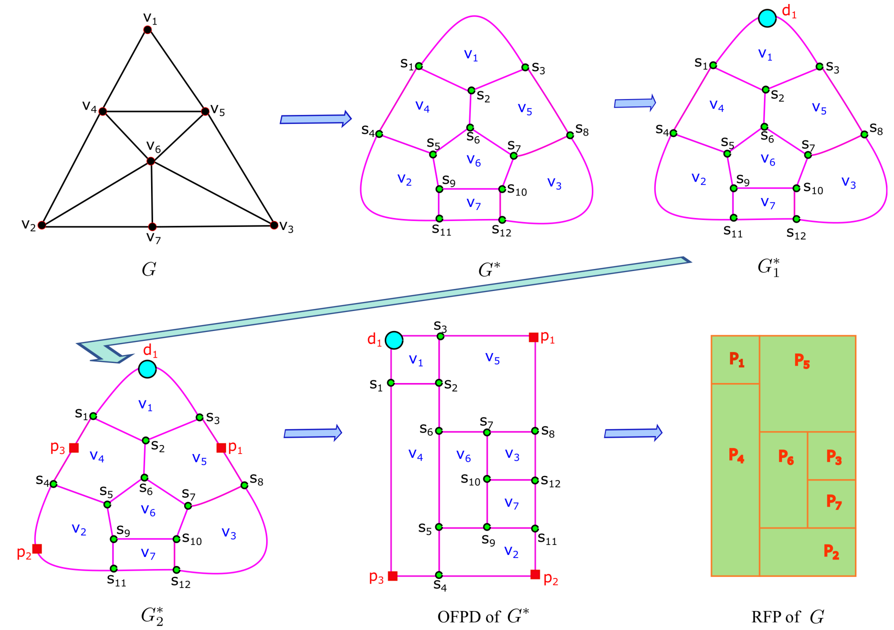

Figure 6. Stepwise computation of a RFP for a plane triangulation G (with no STs and CIPs ≤ 4)

Theorem 2. For any bi-connected plane triangulation G, Algorithm 2 [Generate FP] holds the following statements:

(a) It generates either a RFP or an OFP.

(b) The bends in obtained OFPD = # minimal bad cycles of $G^* + 4 - |d_i's|$.

(c) The bends in FP = # minimal bad cycles $-|d_i's|$.

(d) It is implementable to run in linear-time.

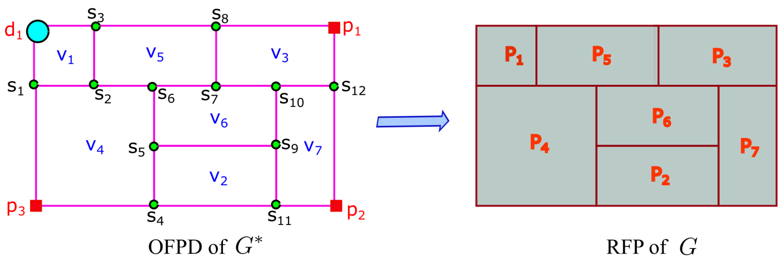

Figure 7. **Another solution for G given in Figure 6**

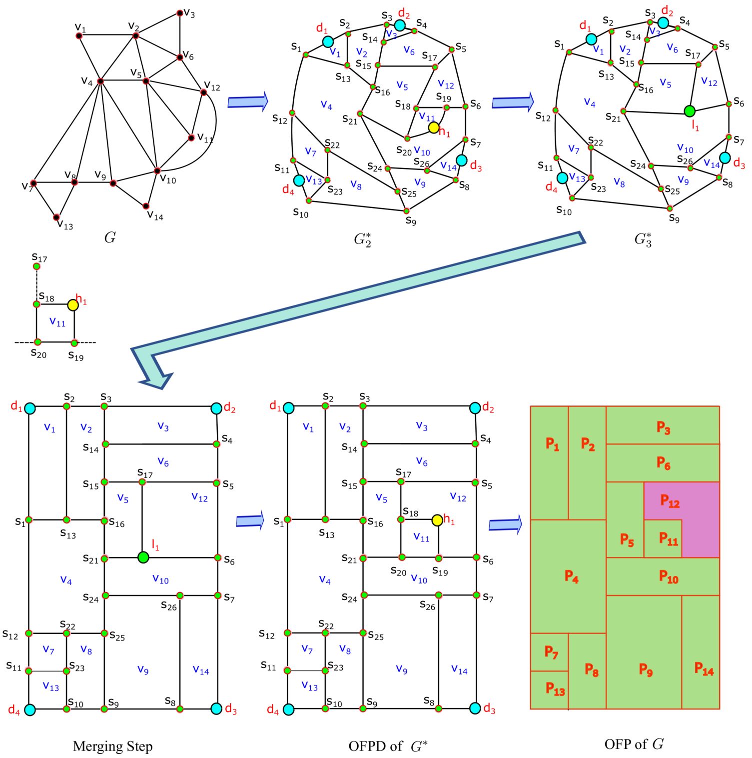

Figure 8. Stepwise computation of an OFP for a plane triangulation G (with STs and 4 CIPs)

Proof. (a) By the definition of RD, it is clear that RD with four designated corners is a RFP. G_2^* has only four dummy vertices which lies on $C_o(G_2^*)$ and if $|h_i's| = 0$ then there are no minimal bad cycles except the ones which lies on $C_o(G^*)$. Hence, by Lemma 1, there exists a RD for G_2^* which is a RFP. If $|h_i's| \neq 0$ in G_2^* (i.e, there exist minimal bad cycles which lies inside of $C_o(G^*)$) then by contracting bad cycles $T_i's$ of G_2^* containing dummy vertices $h_i's$ to $l_i's$, the resulting graph G_3^* has only four dummy vertices which lies on its contour and do not contain

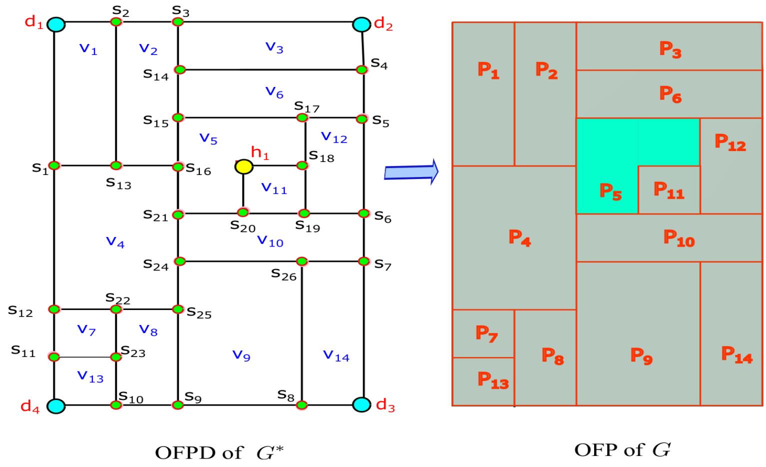

Figure 9. **Another solution for G given in Figure 8**

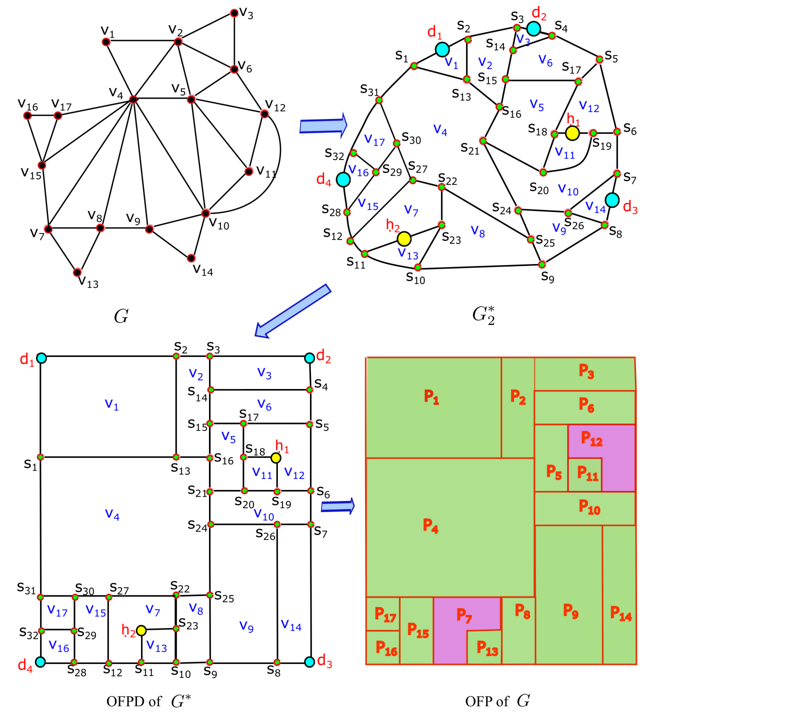

Figure 10. Stepwise computation of an OFP for a plane triangulation G (with STs and 5 CIPs)

any bad cycles. Hence, by Lemma 1 there exists a RD for G_3^* and every T_i is a connected plane graph having four vertices of degree 2 on its contour and all the other vertices are of degree 3 with no bad cycles. Hence, by Lemma 1 there

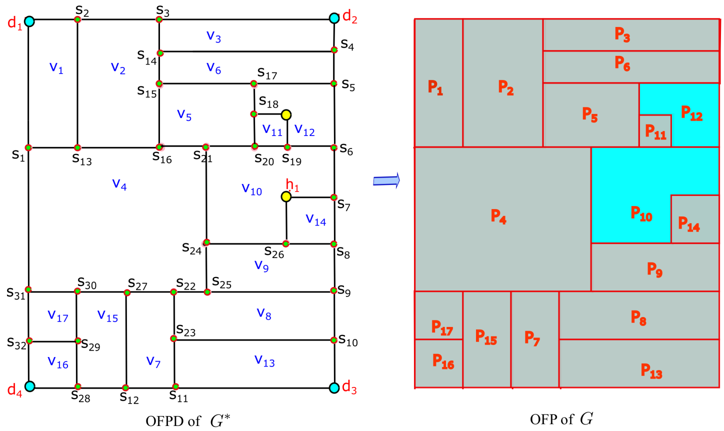

Figure 11. **Another solution for G given in Figure 10**

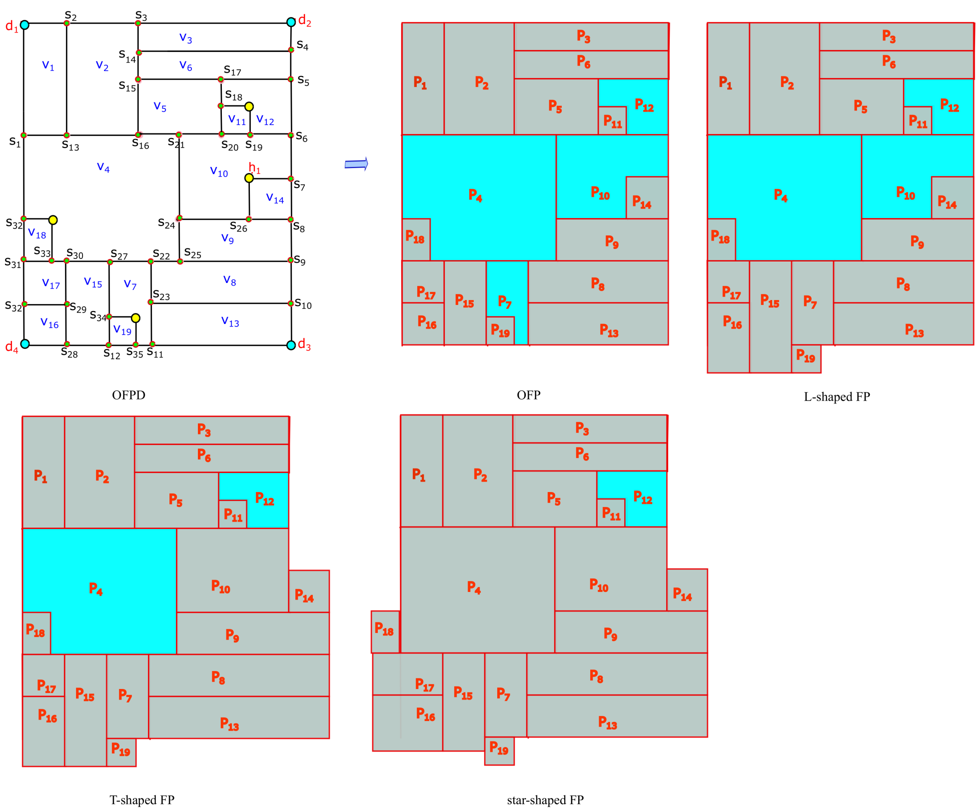

Figure 12. **Different types of FPs corresponding to any OFPD**

exists a RD for every T_i. Merging RDs of G_3^* and $T_i's$ results in an OD of G_2^* with

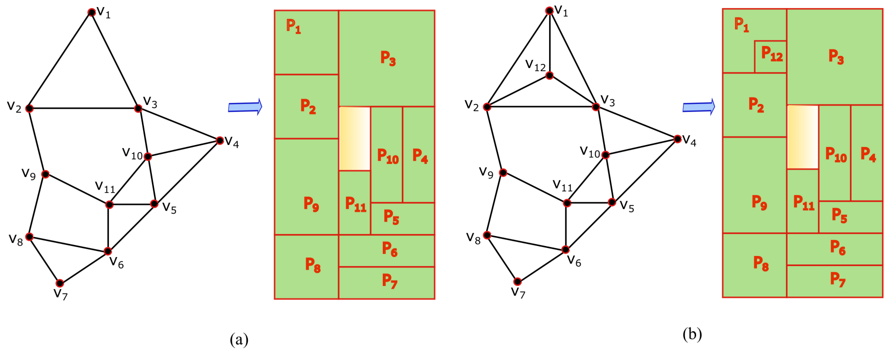

Figure 13. A bi-connected non-triangulated graph G with its possible FP (a) RFP with empty spaces (b) OFP with empty spaces

orthogonal faces, i.e., an OFPD of G^* which is an OFP of G (see Figure 8).

(b) The number of bends in an OFPD of G^*.

Let $C_1, C_2, C_3, \ldots, C_l$ be the minimal bad cycles in G^*. For every C_i where $1 \leq i \leq l$, has exactly one bend according to Fact 1. Furthermore, exactly four bends must appear in an OFPD of G^* corresponding to four dummy vertices of contour of G^* according to Fact 2. Dummy vertices $d_i's$ on $C_o(G^*)$ corresponds to minimal bad cycles. Hence, they are counted twice. Therefore, it results into an OFPD of G^* with exactly # minimal bad cycles $+4 - |d_i's|$ bends.

(c) In OFPD of G^*, only replacing RDs of contracted bad cycles creates a orthogonal shaped module which is because of dummy vertices $h_i's$ (which are bends) and dummy vertices on the contour of G^* does not contribute to the orthogonal shape. Therefore, the number of bends in an OFP of G has (# minimal bad cycles $+4 - |d_i's|$) bends.

(d) The implementation of an Algorithm 2 can be done in linear-time as demonstrated in the explanation given below.

□

2.2.1. Complexity of algorithm

Algorithm 2 [Generate FP] requires three major operations:

1. First Operation: Checking STs and CIPs takes linear-time.
2. Second Operation: EGDR (G^*) of G can be constructed in linear-time.
3. Third Operation: Algorithm 3 is implementable to run in linear-time.

First operation: Checking STs (i.e., non-face cycles of length 3 and if # interior faces $\neq$ # triangles, then there $\exists$ STs) and CIPs ≥ 4 is implementable in linear-time by the concept given in [4].
Second operation: The EGDR (G^*) of G can be constructed in $O(n)$ time because the operations of insertion of a vertex and insertion of an edge takes only $O(1)$ time (i.e., insertion of a vertex in every interior face, where the number of interior faces in a plane graph G are $f = e - n + 1$ and for every edge of $C_o(G)$ in f_e. An edge between two faces is added if they have a common edge and join outer vertices if they have a vertex in common).
Third Operation: By the method similar to Lemma 1, all bad cycles of G^* can be found in $O(n)$ time. RD [28] can also be constructed in $O(n)$ time. Furthermore, all the contraction and merging operations performed in Algorithm 3 (i.e, addition of dummy vertices and replacing minimal bad cycle with a dummy vertex of $d_2(v)$ which do not lie on contour with a single vertex) can be done in $O(n)$ time in total. Therefore, the total time required by Algorithm 3 is $O(n)$.

3. Conclusion and Future Work

It is clear from the definition that every RD is a RFP but not every OD is an OFP. Here, in this study, we establish a constructive approach for an orthogonal floor plan drawing, i.e., an OD which is either an OFP or a RFP. It has been discussed earlier that there are algorithms for constructing FPs for bi-connected near triangulations and near triangulations with an exterior face of length 3 (which are not applicable on examples given in Figure 6 and 8). In this study, we provide a simple generalised linear-time algorithm for constructing a RFP or an OFP (if RFP does not exist), which is applicable to any plane triangulation with an exterior face of any length. Our technique is based on the idea of ODs of triconnected cubic plane graphs. The work given in this paper is not restricted to a single FP corresponding to any bi-connected plane triangulation G but many more topologically distinct FPs can be generated using proposed Algorithm 2 (see examples in Figure 7, 9 and 11).
Our work leads to the following future directions:

1. The work in this paper is focussed on orthogonal drawings which are OFPs. As a future work, this concept can be extended to orthogonal drawings which generate FPs of different shapes, i.e., L-shaped FPs, T-shaped FPs, Star-shaped FPs or Staircase FPs (see example in Figure 12).

2. The work in literature is mostly focussed on FPs for bi-connected triangulated graphs which can be extended to class of bi-connected non- triangulated graphs (see example in Figure 13).

Conflict of interest. The authors declare that they have no conflict of interest.

Data Availability. Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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