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Research Article

Roman domination in signed graphs

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Abstract: Let $S = (G, \sigma)$ be a signed graph. A function $f : V \to \{0, 1, 2\}$ is a Roman dominating function on S if (i) for each $v \in V$, $f(N[v]) = f(v) + \sum_{u \in N(v)} \sigma(uv)f(u) \geq 1$ and (ii) for each vertex v with $f(v) = 0$, there exists a vertex $u \in N^+(v)$ such that $f(u) = 2$. In this paper we initiate a study on Roman dominating function on signed graphs. We characterise the signed paths, cycles and stars that admit a Roman dominating function.

Keywords: Signed graphs, Dominating functions, Roman dominating functions

AMS Subject classification: 05C22

1. Introduction

The concept of signed graphs was introduced by Harary [11] in 1953 as a generalisation of graphs in the context of certain problems in social psychology. Signed graphs have a sign function associated with it according to which each edge is labelled either positive or negative. Formally, a *signed graph* is an ordered triple $S = (V, E, \sigma)$, where $G = (V, E)$ is a simple graph called the *underlying graph* of S and $\sigma : E(G) \to \{-1, 1\}$ is a function called a *signing* of G or the *signature* of S. Whenever we want to mention that G is the underlying graph we may also write $S = (G, \sigma)$. The negative and positive edges are usually represented using dashed and solid lines respectively. A signed graph with every edge positive(negative) is called *all positive(all negative)* signed graph. For any vertex u, $N^+(u) = \{v \in N(u) \mid \sigma(uv) = 1\}$ and $N^-(u) = \{v \in N(u) \mid \sigma(uv) = -1\}$. The positive and negative degree of a vertex u is defined as $d^+(u) = |N^+(u)|$ and $d^-(u) = |N^-(u)|$ respectively, while the degree

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of u in G is $d_G(u) = d^+(u) + d^-(u)$. Signed graphs offer a vast space for researchers to explore theoretical concepts in graphs and several attempts have been made by various authors as we can see in [4, 5, 14–16].
In this paper we initiate a study on Roman dominating functions in the realm of signed graphs. A *Roman dominating function* (RDF) on a graph $G = (V, E)$ is a function $f : V \to \{0, 1, 2\}$ such that every vertex v with $f(v) = 0$ is adjacent to at least one vertex u with $f(u) = 2$. The concept of Roman domination was introduced by Cockayne et al. [10] in the year 2004. Then, more than two hundred papers have been published on this topic, where several new variations such as triple Roman domination [2], maximal Roman domination [3], Total Roman reinforcement [1], Roman 2-domination [7] and double Roman domination [6] have been introduced. For more details on Roman domination and its variations we refer the reader to the recent two book chapters [8, 12] and survey paper [9].

Before proceeding with the definition of Roman domination in signed graphs, we investigate the concept of domination in signed graphs. Acharya [5] examined domination in signed graphs while modelling the Prey-Predator problem. According to him, a set $D \subseteq V$ is called a dominating set of a signed graph $S = (V, E, \sigma)$ if all the vertices of V are either in D or there exists a function $\mu : V \to \{-1, 1\}$ called a marking of S such that all the vertices $u \in V \setminus D$ are adjacent to at least one vertex $v \in D$ such that $\sigma(uv) = \mu(u)\mu(v)$. Jeyalakshmi [14] offered another definition for domination in signed graphs. A set $D \subseteq V$ is called a dominating set of a signed graph $S = (V, E, \sigma)$ if for all $v \in V \setminus D$, $|N^+(v) \cap D| > |N^-(v) \cap D|$. We consider the view point that in any network, existence of positive edges from a set of vertices A to every vertex in another set B ensures that the set A dominates B. Accordingly we present an alternate definition for domination in signed graphs.

Definition 1. Let $S = (V, E, \sigma)$ be a signed graph. A set $D \subseteq V$ is said to be a dominating set in S if for each vertex $v \in V \setminus D$ there exist a vertex $u \in N^+(v) \cap D$. The minimum cardinality among all the dominating sets of S is called the domination number of S, denoted by $\gamma(S)$.

Clearly, if all the edges of S are positive then the above definition reduces to that of the domination in graphs. Now if S is all negative, then the dominating set is trivially V. Recall that a *dominating function* [13] on a graph G is a function $f : V(G) \to \{0, 1\}$ such that for each vertex $v \in V(G)$, $\sum_{u\in N[v]} f(u) \geq 1$. Now we proceed to define the concept of Roman domination in signed graphs.

Definition 2. Let $S = (V, E, \sigma)$ be a signed graph. A function $f : V \to \{0, 1, 2\}$ is a *Roman dominating function(RDF)* on S if it satisfies the following conditions:

(i) for each $v \in V$, $f(N[v]) = f(v) + \sum_{u\in N(v)} \sigma(uv)f(u) \geq 1$ and

(ii) for every vertex v with $f(v) = 0$, there exists a vertex $u \in N^+(v)$ such that $f(u) = 2$.

The value $\omega(f) = \sum_{v\in V} f(v)$ is called the weight of f. The least value of $\omega(f)$ among all the Roman dominating functions f on S is called the *Roman domination number* of S, denoted by $\gamma_R(S)$. A Roman dominating function f with $\omega(f) = \gamma_R(S)$ is called a γ_R-function of S. Note that this definition is equivalent to the definition of Roman domination as defined in [10] when all the edges of S are positive. Further, it is to be noted that a signed graph without any positive edges does not admit any RDF. Therefore throughout our study we consider only pure signed graphs, signed graphs having both positive and negative edges.

2. Preliminary Results

As seen from the definition, not all signed graphs admit RDF. Therefore first we examine some necessary conditions for signed graphs not admitting an RDF. The following observation is useful for further discussions.

Observation 1. If S is a signed graph that admits an *RDF* f and v is a vertex of S such that $N^+(v) = \phi$, then $f(v) \neq 0$. Further, when $d(v) = 2$ and $N(v) = \{u, w\}$, then the following holds.

(i) If $f(v) = 1$, then $f(u) = f(w) = 0$.

(ii) If $f(v) = 2$, then $f(u), f(w) \in \{0, 1\}$ and $f(u)$, $f(v)$ are not simultaneously equal to 1.

Suppose S is a signed graph having a vertex u with $d^-(u) = n - 1$. Then, by using Observation 1, there is no function $f : V \to \{0, 1, 2\}$ on S such that $f(N[u]) \geq 1$ which leads to the following proposition.

Proposition 1. *If S is a signed graph on n vertices containing a vertex u with $d^-(u) = n - 1$, then S does not admit an RDF.*

Next lemma presents yet another class of signed graphs not admitting an RDF.

Lemma 1. *If S is a signed graph having a pair of adjacent vertices u and v such that $N^+(u) = N^+(v) = \phi$, then S does not admit an RDF.*

Proof. If possible assume that S admits an *RDF* f. Since $N^+(u) = N^+(v) = \phi$, by Observation 1, $f(u), f(v) \in \{1, 2\}$. Now consider,

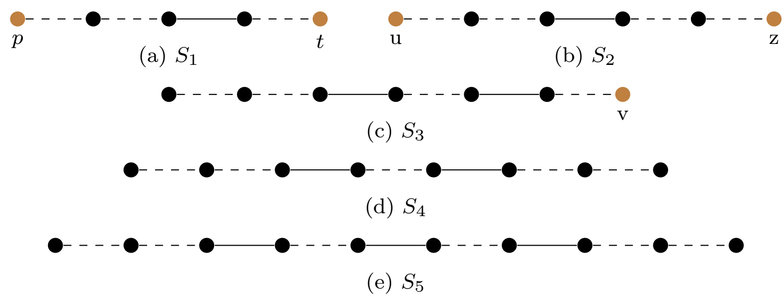
$$f(N[u]) = f(u) + \sum_{v\in N(u)} \sigma(uv)f(v) = f(u) - f(v) - \left(\sum_{x\in N(u)\setminus\{v\}} f(x) \right).$$

If $f(u) = f(v)$ then it follows immediately that, $f(N[u]) = -\sum_{x\in N(u)\setminus\{v\}} f(x) < 1$ showing that f is not an RDF. Now suppose that $f(u) \neq f(v)$ then, either $f(N[u]) =$

$-1 - \left(\sum_{x \in N(u)\setminus\{v\}} f(x)\right)$ or $f(N[v]) = -1 - \left(\sum_{x \in N(v)\setminus\{u\}} f(x)\right)$, proving that f is not an RDF. Since we get contradictions in all the cases we conclude that our assumption is wrong and hence S does not admit an RDF. □

Remark 1. From Lemma 1 it follows that any signed graph admitting an RDF does not contain a path $(v_1, v_2, v_3, \ldots, v_{k-1}, v_k), k \geq 4$ with all its edges negative and $d(v_i) = 2, 2 \leq i \leq k-1$.

Now we proceed to obtain another family of signed graphs not admitting RDF. Let $\mathscr{S} = \{S_1, S_2, S_3, S_4, S_5\}$ be a family of signed paths as shown in Figure 1. In view of Observation 1 and Lemma 1, none of these paths admit an RDF. Now suppose that $S = (G, \sigma)$ is a signed graph containing one of the paths in $\mathscr{S}$ as a proper induced subgraph of G. Note that $d_G(x) = 2$ for all the vertices x of S_i other than the brown vertices. $d_G(t) = d_G(v) = 1$ and the degrees of the vertices p, u and z in G can be any positive integer. We will prove that no signed graph containing any of these paths S_i admit an RDF.



(a) S_1 (b) S_2 (c) S_3 (d) S_4 (e) S_5

Figure 1. **Family $\mathscr{S}$ of signed paths**

Lemma 2. *If $S = (G, \sigma)$ is a signed graph containing any one of the members of the family $\mathscr{S}$ as an induced subgraph, then S does not admit an RDF.*

Proof. If possible assume that $S = (G, \sigma)$ admits an RDF f. We consider 5 different cases.

Case 1. S contains S_1.
Suppose that $S_1 := (p, q, r, s, t)$, where $d_G(t) = 1$, $d_G(p)$ is any positive integer and the remaining vertices are of degree 2 in G. From Observation 1, $f(r) \in \{0, 1\}$ so that $f(s) = 2$ always. Then $f(N[t]) = f(t) - 2 < 1$ for all possible values of $f(t)$, which contradicts the fact that f is an *RDF*.

Case 2. S contains S_2.
Let $S_2 := (u, v, w, x, y, z)$ such that $d_G(u)$ and $d_G(z)$ are any positive integers and

the remaining vertices are of degree 2 in G. By Observation 1, $f(w), f(x) \in \{0,1\}$. If $f(w) = f(x) = 0$, then the vertices w and x are not adjacent to a vertex with label 2 in $N^+(w)$ and $N^+(x)$ respectively. Now suppose that $f(w) = f(x) = 1$, then $f(N[w]) < 1$ and $f(N[x]) < 1$. Therefore assume that $f(w) \neq f(x)$. Then $f(w) = 1, f(x) = 0$ or $f(w) = 0, f(x) = 1$. In the first case x and in the later case w are not adjacent to a vertex with label 2 in $N^+(x)$ and $N^+(w)$ respectively. Thus we arrive at contradictions.

Case 3. S contains S_3.
Assume that $S_3 := (p, q, r, s, t, u, v)$, where $d_G(v) = 1$ and the remaining vertices are of degree 2 in G. Then by following the same argument as in Case 1 $f(r) = 0$ and $f(s) = 2$. Further, $f(t) = 1$. For, if $f(t) = 2$ or $f(t) = 0$, then $f(N[s]) = 0$ or $f(N[t]) = 0$ which is not possible. Similarly, if $f(u) = 0$ or 1 then $f(N[t]) < 1$ in both the cases. Therefore $f(u) = 2$ and $f(N[v]) = f(v) - 2 < 1$ for all possible values of $f(v)$, which is a contradiction.

Case 4. S contains S_4.
Let $S_4 := (p, q, r, s, t, u, v, w)$. By Observation 1, $f(r) = f(u) = 0$ and therefore $f(s) = f(t) = 2$ as f is an RDF. Then $f(N[s]) = 0$, again a contradiction.

Case 5. S contains S_5.
Take $S_5 := (p, q, r, s, t, u, v, w, x, y)$. Then $f(r) = f(w) = 0$ and $f(s) = f(v) = 2$, by the same argument as in Case 4. If $f(t) = 2$ or $f(u) = 2$ then $f(N[s]) < 1$ or $f(N[v]) < 1$. Therefore $f(t), f(u) \neq 2$ and hence $f(u), f(t) \neq 0$. This implies that the only possibility is $f(t) = f(u) = 1$. Then $f(N[t]) < 1$ and $f(N[u]) < 1$ which contradicts the fact that f is an RDF.
Since we get contradictions in all the 5 cases, we conclude that $S = (G, \sigma)$ does not admit any RDF. □

3. Signed Paths and Cycles Admitting RDF

In this section we characterise signed paths and cycles admitting RDF. We use the following definition for further discussion.

Definition 3 ([15]). For a signed path (cycle) S by a *positive (negative) section* we mean a maximal all positive(all negative) subpath of S.

First we present a necessary condition for signed paths P_n to admit an RDF. Since the graphs we consider for the present study contains both negative and positive edges it suffices to consider only the case when $n > 2$. For $n = 3$ it is just a matter of verification to show that P_n admits an RDF. When $n = 4$, there are three signed paths having negative sections of length at most 1. Among these three signed paths the one containing two negative edges as shown in Figure 2 does not admit an RDF. This is because if $f : V(P_4) \to \{0,1,2\}$ is an RDF, by Observation 1, $f(p)$ and $f(s)$ does not take the value 0. Therefore $f(q), f(r) \in \{0,1\}$, in which case $f(N[q]) \leq 1$.

Next we examine signed paths of order more than 4 containing no negative sections of length more than 1.

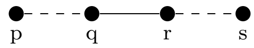


Figure 2. **Signed P_4 not admitting an RDF.**

Theorem 2. *If $P_n(n > 4)$ is a signed path containing no negative sections of length more than 1, then P_n admits an RDF.*

Proof. Let $P_n = v_1v_2v_3\dots v_n$ be a signed path on n vertices. Define a function $f : V \to \{0, 1, 2\}$ on S such that

$$f(v_i) = \begin{cases} 1 & if \quad i = 2, n-1, \\ 2 & \text{otherwise.} \end{cases}$$

Consider the pendant vertex v_1. $f(N[v_1]) = 2 \pm 1 \geq 1$. Similarly, $f(N[v_n]) \geq 1$. Next we consider the support vertex v_2 for which $f(N[v_2]) = 5$ if both the edges incident with v_2 are positive and $f(N[v_2]) = 1$ otherwise. Using the same argument we can show that $f(N[v_{n-1}]) \geq 1$. For all the remaining vertices $v_i; 3 \leq i \leq n-2$, $f(N[v_i]) \in \{1, 2, 3, 5, 6\}$, proving that f is an RDF. □

Remark 2. By using similar arguments used in Theorem 2, it can be proved that signed cycles $C_n(n \geq 3)$ with no negative section of length more than 1 admits an *RDF*.

Now we characterise those paths and cycles admitting an *RDF*.

Theorem 3. *A signed path $(P_n, \sigma), n > 4$ admits an RDF if and only if it satisfies the following conditions:*

(i) *P_n does not contain two adjacent vertices u and v such that $N^+(u) = N^+(v) = \phi$.*

(ii) *P_n does not contain any of the signed paths of the family $\mathscr{S}$.*

Proof. Let $P_n = v_1v_2\cdots v_n$ be a signed path on n vertices.

The necessary part follows from Lemmas 1 and 2.

To prove the sufficiency part, suppose that P_n is a signed path satisfying the given conditions. If P_n does not contain any negative section of length more than 1, then P_n admits an RDF by Theorem 2. Now assume that P_n contains at least one negative

section of length 2. Let (v_{i-1}, v_i, v_{i+1}) be any negative section in P_n. Then by condition (i) in the hypothesis it follows that $3 \leq i \leq n-2$.
Define a function $f : V \to \{0, 1, 2\}$ on P_n such that for any $v_j \in V$, $1 \leq j \leq n$

$$f(v_j) = \begin{cases} 0 & if \quad j = i-1, i+1, \\ 1 & \begin{cases} if & j = i, \\ if & j = i-3 \text{ and } \sigma(v_{i-3}v_{i-2}) = -1 \text{ or} \\ & j = i+3 \text{ and } \sigma(v_{i+2}v_{i+3}) = -1, \\ if & j = 2 \text{ and } \sigma(v_1v_2) = -1 \text{ or} \\ & j = n-1 \text{ and } \sigma(v_{n-1}v_n) = -1, \end{cases} \\ 2 & \text{otherwise.} \end{cases}$$

We claim that f is an RDF. First we observe that $f(v_j) = 0$ only if v_j is a part of a negative section of length two. This implies that, v_j is not a pendant vertex and the other edge incident with v_j is positive. It follows by the definition of f that the neighbour of v_j which is not a part of the negative section has label 2. Therefore every vertex v_j with $f(v_j) = 0$ is adjacent to a vertex with label 2 in $N^+(v_j)$. Now it remains to prove that $f(N[v_j]) \geq 1, 1 \leq j \leq n$. For this we consider three cases:

Case 1. $f(v_j) = 0$.
In this case $j = i-1$ or $i+1$. Further from the definition of f, $f(v_{i-2}) = 2 = f(v_{i+2})$ and $f(v_i) = 1$. Hence $f(N[v_j]) = 0 + 2 - 1 \geq 1$.

Case 2. $f(v_j) = 1$.
Then from the definition of f, there are three possible options for j. The first option is that $j = i$, for which $f(N[v_j]) = 1$. The second possibility is that $j = i-3$ with $\sigma(v_{i-3}v_{i-2}) = -1$. Therefore $\sigma(v_{i-4}v_{i-3}) = 1$. Further, $f(v_{i-4}) = 2$ so that $f(N[v_j]) = 1$. Using the same argument it can be proved that $f(N[v_j]) = 1$ when $j = i+3$ with $\sigma(v_{i+2}v_{i+3}) = -1$. Next we consider the option that $j = 2$ with $\sigma(v_1v_2) = -1$. Then by condition (i), $\sigma(v_2v_3) = 1$. Hence it follows that $f(v_3) = 2$ as S_5 and S_7 are forbidden structures of any signed graph admitting an RDF. Therefore proving that $f(N[v_2]) \geq 1$. Similarly, it can be shown that $f(N[v_{n-1}]) \geq 1$ for $j = n-1$ with $\sigma(v_{n-1}v_n) = -1$.

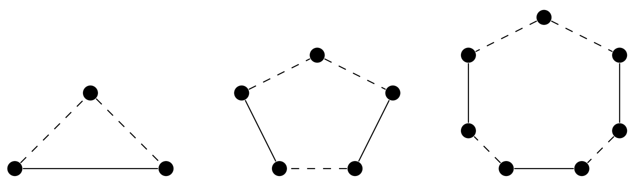
Case 3. $f(v_j) = 2$.
First assume that v_j is a pendant vertex. If the pendant edge containing v_j is positive, then it is obvious that $f(N[v_j]) \geq 1$. Else, the label for the support vertices is 1 so that $f(N[v_j]) \geq 1$.
Now suppose that v_j is not a pendant vertex. If both the edges incident with v_j are positive, then $f(N[v_j]) \geq 1$ for all possible labels of its neighbours. Now if v_j is incident with a positive and a negative edge, then the only possibility where $f(N[v_j]) < 1$ is when $f(v_{j-1}) = 0, f(v_{j+1}) = 2$ and $\sigma(v_{j-1}v_j) = 1$; $\sigma(v_jv_{j+1}) = -1$ or $f(v_{j-1}) = 2, f(v_{j+1}) = 0$ and $\sigma(v_{j-1}v_j) = -1$; $\sigma(v_jv_{j+1}) = 1$. However,

this is not possible. Because, in the former case $f(v_{j-1}) = 0$ which implies that $(v_{j-3}, v_{j-2}, v_{j-1})$ is a negative section and so $f(v_{j+1}) = 1$ by the definition of f. Similarly, in the later case we get $f(v_{j-1}) = 1$. Therefore $f(N[v_j]) \geq 1$ in this case as well.

Thus from the above three cases we have $f(N[v_j]) \geq 1$, for all the vertices v_j of P_n, which completes the proof.

□

Next let us examine signed cycles admitting an RDF. Among signed cycles C_n of order less than 8, it can easily be verified that those given in Figure 3 do not admit an RDF. Further, from Lemmas 1 and 2 it follows that, if C_n contains the paths S_2, S_4 or S_5 of the family $\mathscr{S}$ or has a pair of adjacent vertices with $N^+ = \phi$, then C_n does not admit an RDF. Following a similar argument used in Theorem 3 we have the following result.



(a) C_3 (b) C_5 (c) C_7

Figure 3. **Signed cycles of length less than 8 not admitting any RDF.**

Theorem 4. *A Signed cycle $C_n (n \geq 8)$ admit an RDF if and only if it satisfies the following conditions:*

(i) *C_n does not contain two adjacent vertices u and v such that $N^+(u) = N^+(v) = \phi$.*

(ii) *C_n does not contain any of the signed paths S_2, S_4 and S_5 of the family $\mathscr{S}$.*

4. Signed Stars Admitting RDF

In this section we study the properties of signed stars admitting an RDF. First we present a property of an RDF on a signed star.

Proposition 2. *If $K_{1,n-1}$ is a signed star that admits an RDF f and u is the vertex with $d(u) = n - 1$, then $f(u) \in \{0, 1\}$.*

Proof. Suppose on the contrary that $f(u) = 2$. Then, for any vertex $v \in N^-(u)$, $f(N[v]) = f(v) - 2 < 1$ for all possible values of $f(v)$. This contradicts the fact that f is an *RDF*. Hence our assumption is wrong and therefore $f(u) \in \{0, 1\}$. □

Now we characterize those signed stars admitting an *RDF* in terms of the number of positive and negative edges incident with the central vertex.

Theorem 5. *A signed star $K_{1,n-1}$ admit an RDF if and only if $2d^+(u) > d^-(u)$, where u is the vertex with $d(u) = n-1$.*

Proof. We first prove the sufficiency part. Suppose that $2d^+(u) > d^-(u)$, then $N^+(u) \neq \phi$. Define a function $f : V \to \{0, 1, 2\}$ on S by

$$f(x) = \begin{cases} 0 & if \quad x = u, \\ 1 & if \quad x \in N^-(u), \\ 2 & otherwise. \end{cases}$$

We claim that f is an *RDF*. By definition of f, u is the only vertex with $f(u) = 0$ and it is always adjacent to a vertex $v \in N^+(u)$ with $f(v) = 2$. Moreover, $f(N[u]) = 2d^+(u) - d^-(u) \geq 1$, as $2d^+(u) > d^-(u)$. Now, for every vertex $v \in N^+(u)$, $f(N[v]) = 2$ and for every vertex $w \in N^-(u)$, $f(N[w]) = 1$. Therefore f is an *RDF* on $K_{1,n-1}$.

Conversely, suppose that $K_{1,n-1}$ admits an *RDF* f. We have to prove that $2d^+(u) > d^-(u)$. On the contrary suppose that $2d^+(u) \leq d^-(u)$. Then by Proposition 2 we have $f(u) \in \{0, 1\}$.

Case 1. $f(u) = 0$.
Then,

$$f(N[u]) = \left(\sum_{v \in N^+(u)} f(v) \right) - \left(\sum_{w \in N^-(u)} f(w) \right) \leq 2d^+(u) - d^-(u) \leq 0,$$

which is not possible since f is an *RDF* and hence we get a contradiction.

Case 2. $f(u) = 1$.
In this case, since $f(u) = 1$, we have $f(w) = 2$ for each $w \in N^-(u)$. For, if $f(w) = 0$ or 1 then $f(N[w]) = f(w) - 1 < 1$. Therefore $f(w) = 2$ for each $w \in N^-(u)$ and $\sum_{w \in N^-(u)} f(w) = 2d^-(u)$. Now, since by assumption $2d^+(u) \leq d^-(u)$, we have $d^+(u) < d^-(u)$. This implies that $d^+(u) - d^-(u) \leq -1$. By computing $f(N[u])$ we get,

$$\begin{aligned} f(N[u]) &= 1 + \left(\sum_{v \in N^+(u)} f(v) \right) - \left(\sum_{w \in N^-(u)} f(w) \right) \\ &\leq 1 + 2d^+(u) - 2d^-(u) \\ &= 1 + 2(d^+(u) - d^-(u)) \leq 1 - 2 = -1 \end{aligned}$$

which is a contradiction to the fact that f is an RDF.

Since we obtain contradictions in both the cases we conclude that our assumption is wrong and therefore $2d^+(u) > d^-(u)$.

□

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