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Research Article

A second-order corrector wide neighborhood infeasible interior-point method for linear optimization based on a specific kernel function

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Abstract: In this paper, we present a second-order corrector infeasible interior-point method for linear optimization in a large neighborhood of the central path. The innovation of our method is to calculate the predictor directions using a specific kernel function instead of the logarithmic barrier function. We decompose the predictor direction induced by the kernel function to two orthogonal directions of the corresponding to the negative and positive component of the right-hand side vector of the centering equation. The method then considers the new point as a linear combination of these directions along with a second-order corrector direction. The convergence analysis of the proposed method is investigated and it is proved that the complexity bound is $\mathcal{O}(n^{\frac{5}{4}} \log \varepsilon^{-1})$.

Keywords: Linear optimization, predictor-corrector methods, wide neighborhoods, polynomial complexity

AMS Subject classification: 90C51, 90C05

1. Introduction

Interior point methods (IPMs) which started by Karmakar's paper in 1984 [7] have attracted much attention because of their power for solving linear optimization (LO). These methods were extensively used to obtain strong theoretical complexity results for LO. Among them, the primal-dual IPMs gain much more attention and the central pathway independently proposed by Megiddo [18] and Sonnevend [24] plays a crucial role in these methods. Actually, the goal of primal-dual IPMs is to follow the central

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path and to compute an interior point as an ε-approximate of the optimal solution. For more details we refer the interested reader to the monographs of [23, 28, 29]. As can be seen from the literature, one classification of the primal-dual path-following IPMs can be based on step length; e.g., short update (small neighborhood) and large update (wide neighborhood) methods. From the point of view of theory, the small neighborhood methods give the complexity results better, while the large-step versions perform better in practice. This suggests that there is a gap between theory and practice. To reduce gap mentioned, Peng et al. [21] the first introduced the interior-point algorithms based on the idea of the self-regular functions. Wang et al. [27] presented a full-Newton step feasible IPM for $P_*(\kappa)$-LCP and obtained the currently best known iteration bound for small-update methods. Wang et al. [26] proposed an interior point algorithm and improved the complexity bound of IPMs for SDO using the Nesterov and Todd (NT) direction. Another classification is related to the feasibility of the iterates. In this case, we can talk about feasible and infeasible interior-point algorithms [23, 28, 29]. We can also meet with predictor-corrector IPMs which seems to be, in the theory and practice, the most powerful among these types of algorithms [19, 20]. Darvay [4] published a predictor-corrector interior-point algorithm for LO based on the algebraic equivalent transformation of the central path and used the function $\psi(t) = \sqrt{t}, t > 0$ in order to determine of the transformed central path. Some variants and extensions of this algorithm can be found in [8–10, 25].

In 2004, Ai [1] proposed a new neighborhood of the central path for LO that was wider from the existing neighborhoods. Based on this neighborhood, Ai and Zhang [2] introduced a new path-following interior-point algorithm for LCP and proved that their algorithm has the $\mathcal{O}(\sqrt{n}L)$ iteration complexity bound. later on, this work attracted the attention of a number of researchers; e.g., Li and Terlaky [15], Feng and Fang [6] and Potra [22] generalized the Ai-Zhang idea to SDO and $P_*(\kappa)$-LCP, respectively. Feng [5] extended the Ai-Zhang technique for solving LCP to second-order cone optimization (SOCO). Liu and Liu [16] proposed the first wide neighborhood second-order corrector IPM with the same complexity as small neighborhood IPMs for SDO. Kheirfam and Chitsaz [12] proposed a second-order corrector interior-point algorithm for solving $P_*(\kappa)$-LCP based on the Ai-Zhang's idea [2] and proved that the algorithm meets the best known theoretical complexity bound. Kheirfam and Mohammadi-Sangachin [14] proposed a new predictor-corrector IPM for SDO in which their algorithm computes two corrector directions in addition to the Ai-Zhang directions.

All the algorithms mentioned so far require a strictly feasible starting point. For most problems such a starting point is difficult to find. In this case, an infeasible IPM (IIPM) is suggested. Liu et al. [17] devised a new infeasible-interior-point algorithm based on a wide neighborhood for symmetric optimization (SO). Based on the wide neighborhood and a commutative class of search directions, Kheirfam [11] investigated a predictor-corrector infeasible interior-point algorithm for SDO and proved that the complexity of the proposed algorithm is $\mathcal{O}(n^{5/4}\log\varepsilon^{-1})$.

It is worth noting that all the aforementioned wide neighborhood interior point algorithms use the classical logarithmic barrier function to obtain the search directions.

Kheirfam and Haghighi [13] for the first time introduced a wide neighborhood interior-point algorithm based on the kernel function for LO. In fact, the search direction is based on the kernel function. Motivated by these observations, we propose a second-order corrector wide neighborhood infeasible interior-point algorithm for LO. Our algorithm decomposes the predictor directions induced by the kernel function to two orthogonal directions corresponded to the negative and positive parts of the right-hand side vector of the centering equation. Using the information of the negative directions, the algorithm computes a second-order corrector direction. The corrector is multiplied by the square of the twice the step size of the negative directions in the expression of the new iterate. We establish polynomial-time convergence of the proposed algorithm and derive the iteration complexity bound. Notably, this is the first second-order corrector wide neighborhood infeasible interior-point algorithm based on the kernel function for LO.

The outline of the paper is as follows. In Section 2, we introduce the LO, the concept of its central path and the search directions based on the kernel function. In Section 3, we give the systems defining the negative and positive search directions and the second-order corrector direction. Then, we present our new algorithm. Section 4 is devoted to the analysis of the proposed algorithm and we compute the iteration complexity for the algorithm. Finally, some concluding remarks are given in Section 5.

Notations. The following notations are used throughout the paper. $R^{m\times n}$ denotes the set of all $m\times n$ matrices, whereas R^n is the n-dimensional Euclidean space. The Euclidean norm of $v\in R^n$ is denoted by $\|v\|$. Let $e=[1,\dots,1]^T$ denotes the n-dimensional all-one vector and let $R^n_+=\{x\in R^n : x\geq 0\}$ and $R^n_{++}=\{x\in R^n : x>0\}$. If $x,s\in R^n$, then xs denotes Hadamard product of two vector x and s, i.e. $xs=[x_1s_1,,\dots,x_ns_n]^T$. Moreover, $\mathrm{diag}(x)$ is a diagonal matrix, which contains on his main diagonal the element of x in the original order. Let $x^+=[x_1^+,\dots,x_n^+]^T$ and $x^-=[x_1^-,\dots,x_n^-]^T$, where $x_i^+=\max\{x_i,0\}$ and $x_i^-=\min\{x_i,0\}$, for each $i=1,\dots,n$, respectively.

2. Preliminaries

Consider the problem pair (P) and (D) as follows:

$$\min\{c^Tx : Ax=b,\ x\geq 0\}, \qquad (P)$$

and

$$\max\{b^Ty : A^Ty+s=c,\ s\geq 0\}, \qquad (D)$$

where $b\in R^m, c\in R^n, A\in R^{m\times n}$. It is assumed that $\mathrm{rank}(A)=m$. By $\mathcal{F}(\mathcal{F}^0)$ we denote the set of all feasible solutions (strictly feasible solutions) of the primal-dual pair of problems (P) and (D). Without loss of generality, we may assume that $\mathcal{F}^0\neq\emptyset$.

The optimal conditions for the problem pair (P) and (D) are

$$\begin{aligned} Ax = b, &\quad x \geq 0, \\ A^T y + s = c, &\quad s \geq 0, \\ xs = 0. & \end{aligned} \tag{1}$$

The key idea of primal-dual IPMs is to use the parameterized equation $xs = \mu e$ instead the complementarity condition $xs = 0$, where $\mu > 0$. Thus, the system (1) becomes

$$\begin{aligned} Ax = b, &\quad x > 0, \\ A^T y + s = c, &\quad s > 0, \\ xs = \mu e. & \end{aligned} \tag{2}$$

By assumption $\mathcal{F}^0 \neq \emptyset$, it is proved that the system (2) has a unique solution $(x(\mu), y(\mu), s(\mu))$ for each $\mu > 0$. The set of all such solutions, denoted by $\mathcal{C} := \{(x(\mu), y(\mu), s(\mu)) : \mu > 0\}$, is called the central path (see [18, 24]). The central path converges for $\mu \downarrow 0$ to the optimal solution of the initial problem.
Applying Newton's method to system (2) gives the following system for search direction $(\Delta x, \Delta y, \Delta s)$:

$$\begin{aligned} A\Delta x &= r_p, \\ A^T \Delta y + \Delta s &= r_d, \\ x\Delta s + s\Delta x &= \tau\mu e - xs, \end{aligned} \tag{3}$$

where $\tau \in (0, 1)$ is called centering parameter, $r_p = b - Ax$ and $r_d = c - s - A^T y$ are the residual vectors at (x, y, s). One easy finds

$$\begin{aligned} x\Delta s + s\Delta x &= -\sqrt{\tau\mu xs}\Big(\sqrt{\frac{xs}{\tau\mu}} - \sqrt{\frac{\tau\mu e}{xs}}\Big) \\ &= -\sqrt{\tau\mu xs}\nabla\Psi\Big(\sqrt{\frac{xs}{\tau\mu}}\Big), \end{aligned} \tag{4}$$

where $\Psi(t) = \sum_{i=1}^n \psi(t_i)$, $\psi(t_i) = \frac{t_i^2 - 1}{2} - \log(t_i)$ with $t_i = \sqrt{\frac{x_i s_i}{\tau\mu}}$ and $t \in R^n$. ψ is so-called a kernel function, i.e., $\psi(1) = \psi^{'}(1) = 0, \psi^{''}(t_i) > 0, \forall t_i > 0$ and $\lim_{t_i \to 0^+} \psi(t_i) = \lim_{t_i \to +\infty} \psi(t_i) = +\infty$.
Here, we take the $\psi(t_i) = (t_i - \frac{1}{t_i})^2$ kernel function [3]. In this way, by invoking (4), the system (3) becomes

$$\begin{aligned} A\Delta x &= r_p, \\ A^T \Delta y + \Delta s &= r_d, \\ x\Delta s + s\Delta x &= (xs)^{-1}\big(\tau^2\mu^2 e - (xs)^2\big). \end{aligned} \tag{5}$$

3. An infeasible-interior-point algorithm

We propose a second-order corrector infeasible-interior-point algorithm for solving the problem pair (P) and (D). The algorithm solve the following two systems

$$\begin{aligned} A\Delta x_- &= r_p, \\ A^T\Delta y_- + \Delta s_- &= r_d, \\ x\Delta s_- + s\Delta x_- &= (xs)^{-1}(\tau^2\mu^2 e - (xs)^2)^-, \end{aligned} \tag{6}$$

and

$$\begin{aligned} A\Delta x_+ &= 0, \\ A^T\Delta y_+ + \Delta s_+ &= 0, \\ x\Delta s_+ + s\Delta x_+ &= (xs)^{-1}(\tau^2\mu^2 e - (xs)^2)^+. \end{aligned} \tag{7}$$

to obtain the predictor directions $(\Delta x_-, \Delta y_-, \Delta s_-)$ and $(\Delta x_+, \Delta y_+, \Delta s_+)$, while the corrector direction $(\Delta x^c, \Delta y^c, \Delta s^c)$ is computed by the following system:

$$\begin{aligned} A\Delta x^c &= 0, \\ A^T\Delta y^c + \Delta s^c &= 0, \\ x\Delta s^c + s\Delta x^c &= -\Delta x_-\Delta s_-. \end{aligned} \tag{8}$$

Finally, the new iterate is considered as follows

$$\begin{aligned} x(\alpha) &:= x + \Delta x(\alpha) = x + \frac{\alpha_1}{2}\Delta x_- + \alpha_2\Delta x_+ + \alpha_1^2\Delta x^c, \\ s(\alpha) &:= s + \Delta s(\alpha) = s + \frac{\alpha_1}{2}\Delta s_- + \alpha_2\Delta s_+ + \alpha_1^2\Delta s^c. \end{aligned}$$

Using the above two equations and invoking the systems (6), (7) and (8), we can write

$$\begin{aligned} x(\alpha)s(\alpha) = xs &+ \frac{\alpha_1}{2}((xs)^{-1}(\tau^2\mu^2 e - (xs)^2)^+) \\ &+\alpha_2((xs)^{-1}(\tau^2\mu^2 e - (xs)^2)^+) + \frac{\alpha_1^3}{2}(\Delta s_-\Delta x^c + \Delta x_-\Delta s^c) \\ &+\frac{\alpha_1\alpha_2}{2}(\Delta x_+\Delta s_- + \Delta s_+\Delta x_-) + \alpha_2\alpha_1^2(\Delta s_+\Delta x^c + \Delta s^c\Delta x_+) \\ &+\alpha_2^2(\Delta x_+\Delta s_+) + \alpha_1^4(\Delta x^c\Delta s^c). \end{aligned} \tag{9}$$

Inspired by Ai and Zhang [2], we define the following wide neighborhood of the infeasible central path for LO:

$$\mathcal{N}(\tau,\beta) = \left\{(x,y,s) \in R^n_{++} \times R^m \times R^n_{++} : \left\|(\tau\mu e - xs)^+\right\| \leq \beta\tau\mu\right\},$$

where $\beta, \tau \in (0,1)$ are given constants. One has $(xs)_i \geq (1-\beta)\tau\mu$ for $(x,y,s) \in \mathcal{N}(\tau,\beta)$. Note that the algorithm will be restrict the iterates to the $\mathcal{N}(\tau,\beta)$ wide neighborhood.

Now, we outline our new second-order corrector infeasible-interior-point algorithm as follows.

Algorithm 1 : second − order corecctor infeasible interior − point algorithm

Input: an accuracy parameter $\varepsilon > 0$, neighborhood parameters $0 < \beta \leq \frac{1}{2}, 0 < \tau \leq \frac{1}{4}$ and an initial point $(x^0, y^0, s^0) \in \mathcal{N}(\tau,\beta)$ with $\mu_0 = \frac{(x^0)^T s^0}{n}$.

For $k = 0, 1, \cdots,$

Step 1: If $\mu_k \leq \varepsilon$, then stop.

Step 2: Compute the predictor directions $(\Delta x_-, \Delta y_-, \Delta s_-)$ and $(\Delta x_+, \Delta y_+, \Delta s_+)$ by solving (6) and (7), respectively.

Step 3: Compute the corrector directions $(\Delta x^c, \Delta y^c, \Delta s^c)$ by solving (8).

Step 4: Find the largest step size $0 < \alpha^k := (\alpha_1^k, \alpha_2^k)$, such that

$$\mu(\alpha) \leq \Big(1 - \frac{\alpha_1^k}{10}\Big)\mu, \tag{10}$$

$$x(\alpha^k)^T s(\alpha^k) \geq \Big(1 - \frac{\alpha_1^k}{2}\Big)(x^k)^T s^k, \tag{11}$$

$$\big(x(\alpha^k), y(\alpha^k), s(\alpha^k)\big) \in \mathcal{N}(\tau,\beta). \tag{12}$$

Step 4: Let $(x^{k+1}, y^{k+1}, s^{k+1}) := (x(\alpha^k), y(\alpha^k), s(\alpha^k))$ and $\mu_{k+1} = \frac{(x^{k+1})^T s^{k+1}}{n}$.

Step 5: Set $k := k+1$ and go to *Step 1*.

Let the iterate (x^k, y^k, s^k) be generated by Algorithm 1, and when the residual vectors r_p and r_d are evaluated at the point $(x,y,s) = (x^k, y^k, s^k)$, we denote them by r_p^k and r_d^k. Moreover, we introduce the scalar quantity

$$\nu_k = \Big(1 - \frac{\alpha_1^k}{2}\Big)\nu_{k-1} = \prod_{i=0}^{k-1}\Big(1 - \frac{\alpha_1^i}{2}\Big), \quad \nu_0 = 1.$$

In this way, one easily checks that $r_p^k = \nu_k r_p^0$ and $r_d^k = \nu_k r_d^0$. From this discussion it concludes that $\nu_k = \frac{\|r_p^k\|}{\|r_p^0\|} = \frac{\|r_d^k\|}{\|r_d^0\|}$. This means that ν_k shows the relative infeasibility of (x^k, y^k, s^k). Therefore, at each iteration, we maintain the following condition

$$(x^k)^T s^k \geq \Big(1 - \frac{\alpha_1^k}{2}\Big)(x^{k-1})^T s^{k-1}) \geq \nu_k (x^0)^T s^0, \tag{13}$$

which guarantees that the infeasibility tends to zero while $x^T s$ approaches to zero. Suppose ρ_* be a positive constant such that for some optimal solution (x^*, y^*, s^*), we have $\rho_* = \|x^*\|_\infty + \|s^*\|_\infty$. The starting point (x^0, y^0, s^0) is chosen so that

$$x^0 = s^0 = \rho_* e > 0. \tag{14}$$

From the equality (14) and the definition of ρ_*, it follows that $x^0 - x_* \geq 0$ and $s^0 - s_* \geq 0$. In addition, by direct calculation we obtain

$$\rho_* = \frac{n(\|x^*\|_\infty + \|s^*\|_\infty)}{n} \geq \frac{e^T x^* + e^T s^*}{n}. \tag{15}$$

4. Analysis of the algorithm

For the sake of simplicity, we define the following notations

$$v = \sqrt{xs}, \quad dx_- = \frac{v\Delta x_-}{x}, \quad ds_- = \frac{v\Delta s_-}{s}, \quad dx_+ = \frac{v\Delta x_+}{x},$$
$$ds_+ = \frac{v\Delta s_+}{s}, \quad dx^c = \frac{v\Delta x^c}{x}, \quad ds^c = \frac{v\Delta s^c}{s}.$$

Due to the above notations, we scale the systems (6), (7) and (8) as follows

$$\begin{aligned} \bar{A} dx_- &= r_p, \\ \bar{A}^T \Delta y_- + ds_- &= v s^{-1} r_d, \\ dx_- + ds_- &= \tfrac{\tau\mu e + v^2}{v^3} (\tau\mu e - v^2)^-, \end{aligned} \tag{16}$$

$$\begin{aligned} \bar{A} dx_+ &= 0, \\ \bar{A}^T \Delta y_+ + ds_+ &= 0, \\ dx_+ + ds_+ &= \tfrac{\tau\mu e + v^2}{v^3} (\tau\mu e - v^2)^+, \end{aligned} \tag{17}$$

and

$$\begin{aligned} \bar{A} dx^c &= 0, \\ \bar{A}^T \Delta y^c + ds^c &= 0, \\ ds^c + dx^c &= -v^{-1} dx_- ds_-, \end{aligned} \tag{18}$$

where $\bar{A} = AV^{-1}X$ with $V = \mathrm{diag}(v)$ and $X = \mathrm{diag}(x)$. From the first two equations of (17) and (18) one has $(dx_+)^T(ds_+) = (dx^c)^T(ds^c) = (dx_+)^T ds^c = 0$ and $(dx^c)^T(ds_+) = 0$. Using (9), together with (16) and (17), we can write

$$\begin{aligned} \mu(\alpha) = \frac{x(\alpha)^T s(\alpha)}{n} &= \mu + \frac{\alpha_1}{2n} e^T \left(\frac{\tau\mu e + v^2}{v^2} (\tau\mu e - v^2)^- \right) \\ &+ \frac{\alpha_2}{n} e^T \left(\frac{\tau\mu e + v^2}{v^2} (\tau\mu e - v^2)^+ \right) + \frac{\alpha_1^3}{2n} \big((ds_-)^T (dx^c) + (dx_-)^T (ds^c) \big) \\ &+ \frac{\alpha_1 \alpha_2}{2n} \big((dx_+)^T (ds_-) + (ds_+)^T (dx_-) \big). \end{aligned} \tag{19}$$

The following lemma is key to our analysis.

Lemma 1. ([30]) *Let $u, v \in R^n$ and $p = (x^{-1}s)^{\frac{1}{2}}$, then*

$$\|uv\|_1 \leq \|pu\|\|p^{-1}v\| \leq \frac{1}{2}\big(\|pu\|^2 + \|p^{-1}v\|^2\big).$$

We now cite the following two lemmas from [13].

Lemma 2. *Let $(x, y, s) \in \mathcal{N}(\tau, \beta)$. Then*

(i) $e^T\big(\frac{\tau\mu e + v^2}{v^2}(\tau\mu e - v^2)^-\big) \leq -(1-\tau)n\mu$.

(ii) $e^T\big(\frac{\tau\mu e + v^2}{v^2}(\tau\mu e - v^2)^+\big) \leq \frac{2-\beta}{1-\beta}\sqrt{n}\beta\tau\mu$.

Lemma 3. *If $(x, y, s) \in \mathcal{N}(\tau, \beta)$ and $\beta \leq \frac{1}{2}$, then*

(i) $\big\|\frac{\tau\mu e + v^2}{v^3}(\tau\mu e - v^2)^-\big\|^2 \leq 4n\mu$.

(ii) $\big\|\frac{\tau\mu e + v^2}{v^3}(\tau\mu e - v^2)^+\big\|^2 \leq \frac{\beta^2(2-\beta)^2}{(1-\beta)^3}\tau\mu$.

In order to get an upper bound for $\|dx_-\|\|ds_-\|$, we first prove the following lemma.

Lemma 4. *Let (dx_-, dy_-, ds_-) be a solution of the system (16) and $p = (x^{-1}s)^{\frac{1}{2}}$. Moreover, let (x^0, y^0, s^0) be defined as in (14). Then*

$$(dx_-)^T(ds_-) \geq -\xi t - n\mu,$$

where

$$\xi = \nu\big(\|p^{-1}(s^0 - s^*)\| + \|p(x^0 - x^*)\|\big) \text{ and } t = \big(\|dx_-\|^2 + \|ds_-\|^2\big)^{\frac{1}{2}}.$$

Proof. Considering the first two equations of the system (16) and using the fact that (x^*, y^*, s^*) is an optimal solution, we can write

$$\begin{aligned}0 = \bar{A}dx_- - r_p = AXV^{-1}dx_- - \nu r_p^0 &= AXV^{-1}dx_- - \nu(b - Ax^0)\\ &= A\big(XV^{-1}dx_- - \nu(x^* - x^0)\big).\end{aligned}$$

In a quite similar way, one has

$$A^T\big(XV^{-1}\Delta y_- - vs^{-1}\nu(y^* - y^0)\big) + ds_- - vs^{-1}\nu(s^* - s^0) = 0.$$

From the above two equations, it follows that

$$(XV^{-1}dx_- - \nu(x^* - x^0))^T(ds_- - vs^{-1}\nu(s^* - s^0)) = 0.$$

By rearranging, we have

$$\begin{aligned}XV^{-1}dx_-^Tds_- = \nu xs^{-1}(s^*-s^0)^Tdx_- + \nu(x^*-x^0)^Tds_-\\ -\nu^2vs^{-1}(x^0-x^*)^T(s^0-s^*),\end{aligned}$$

and then one finds

$$\begin{aligned}dx_-^Tds_- &= \nu vs^{-1}(s^*-s^0)^Tdx_- + \nu x^{-1}v(x^*-x^0)^Tds_-\\ &\qquad -\nu^2(x^*-x^0)^T(s^*-s^0)\\ &= \nu(s^*-s^0)^T\Delta x_- + \nu(x^*-x^0)^T\Delta s_- - \nu^2(x^0-x^*)^T(s^0-s^*)\\ &\geq -\nu\|p\Delta x_-\|\|p^{-1}(s^0-s^*)\| - \nu\|p^{-1}\Delta s_-\|\|p(x^0-x^*)\|\\ &\qquad -\nu^2(x^0-x^*)^T(s^0-s^*)\\ &\geq -\nu\big(\|p^{-1}(s^0-s^*)\| + \|p(x^0-x^*)\|\big)\\ &\qquad \times\sqrt{\|p\Delta x_-\|^2+\|p^{-1}\Delta s_-\|^2} - \nu^2(x^0-x^*)^T(s^0-s^*)\\ &= -\xi t - \nu^2(x^0-x^*)^T(s^0-s^*) \geq -\xi t - n\mu.\end{aligned}$$

Here the first inequality follows from the Cauchy-Schwarz inequality and the second inequality is due to the following inequality, for any $a,b,c,d \in R^n$

$$\|a\|\|b\| + \|c\|\|d\| \leq [\|b\|+\|d\|][\|a\|^2+\|c\|^2]^{\frac{1}{2}},$$

and the last inequality follows by

$$\begin{aligned}\nu^2(x^*-x^0)^T(s^*-s^0) &= \nu^2\big((x^0)^T(s^0) - (x^0)^T(s^*) - (x^*)^T(s^0)\big)\\ &\leq \nu^2((x^0)^T(s^0)) \leq \nu(x^Ts) \leq n\mu.\end{aligned}$$

Thus we have completed the proof. □

Lemma 5. *If $(x,y,s) \in \mathcal{N}(\tau,\beta)$, then*

$$\|dx_+\|^2 + \|ds_+\|^2 \leq \frac{\beta^2(2-\beta)^2}{(1-\beta)^3}\tau\mu.$$

Proof. Taking the squared-norm on both sides of the third equation of (17), remembering that $dx_+^Tds_+ = 0$, implies that

$$\begin{aligned}\|dx_+ + ds_+\|^2 = \|dx_+\|^2 + \|ds_+\|^2 &= \Big\|\frac{\tau\mu e + v^2}{v^3}(\tau\mu e - v^2)^+\Big\|^2\\ &\leq \frac{\beta^2(2-\beta)^2}{(1-\beta)^3}\tau\mu,\end{aligned}$$

where the inequality is obtained from Lemma 3(ii). Thus the proof is completed. □

Lemma 6. *If* $(x, y, s) \in \mathcal{N}(\tau, \beta)$, *then*

$$\|dx_-\|^2 + \|ds_-\|^2 \le \omega^2 n^2 \mu,$$

where $\omega = \frac{3}{\sqrt{(1-\beta)\tau}} + \sqrt{\frac{9}{(1-\beta)\tau} + \frac{6}{n}} \ge 6.$

Proof. Using (16) and Lemma 3(i), it follows that

$$t^2 = \|dx_-\|^2 + \|ds_-\|^2 \le -2(dx_-)^T(ds_-) + 4n\mu \le 2\xi t + 6n\mu,$$

where the second inequality is due to Lemma 4. This implies

$$0 \le t \le \xi + \sqrt{\xi^2 + 6n\mu}. \tag{20}$$

From proof of lemma 4.6 in [11], we have

$$\xi \le \frac{3n\mu}{\sqrt{(1-\beta)\tau\mu}}.$$

Substitution of this bound into (20) yields

$$t^2 \le \left(\frac{3n\mu}{\sqrt{(1-\beta)\tau\mu}} + \sqrt{\frac{9n^2\mu^2}{(1-\beta)\tau\mu} + 6n\mu} \right)^2 \le \omega^2 n^2 \mu,$$

where $\omega = \frac{3}{\sqrt{(1-\beta)\tau}} + \sqrt{\frac{9}{(1-\beta)\tau} + \frac{6}{n}} \ge 6$. The proof is completed. □

Based on the results in Lemmas 5 and 6 and the inequality of arithmetic and geometric means, we can straightforwardly prove the following corollary.

Corollary 1. *Suppose that* $(x, y, s) \in \mathcal{N}(\tau, \beta)$. *Then*

(i) $\|dx_+\|\|ds_+\| \le \frac{\beta^2(2-\beta)^2}{2(1-\beta)^3}\tau\mu$.

(ii) $\|dx_-\|\|ds_-\| \le \frac{\omega^2 n^2 \mu}{2}$.

Lemma 7. *Let* $(x, y, s) \in \mathcal{N}(\tau, \beta)$ *and* $p = (x^{-1}s)^{\frac{1}{2}}$, *then*

$$\|dx^c\|\|ds^c\| \le \frac{\omega^4 n^4 \mu}{8(1-\beta)\tau}, \quad \text{where } \omega \ge 6.$$

Proof. If we take the squared-norm of the third equation of (18) and use the orthogonality property of the vectors dx^c and ds^c, then we obtain

$$\|dx^c\|^2 + \|ds^c\|^2 = \|v^{-1}(dx_- ds_-)\|^2 \leq \|v^{-1}\|_\infty^2 \|dx_- ds_-\|^2 \leq \frac{\omega^4 n^4 \mu}{4(1-\beta)\tau}, \tag{21}$$

where the last inequality follows from the fact that $(x, y, s) \in \mathcal{N}(\tau, \beta)$ and Corollary 1(ii). Finally, using the inequality of arithmetic and geometric means and the above inequality, we obtain the inequality in lemma.

□

The next result is a straightforward consequence of Lemmas 5, 6 and the inequality (21).

Corollary 2. *Let $(x, y, s) \in \mathcal{N}(\tau, \beta)$ and $p = (x^{-1}s)^{\frac{1}{2}}$, then*

(i) $\|dx_-\|\|ds_+\| \leq \frac{\beta(2-\beta)}{(1-\beta)^{\frac{3}{2}}}\sqrt{\tau} n\omega\mu$,

(ii) $\|ds_-\|\|dx_+\| \leq \frac{\beta(2-\beta)}{(1-\beta)^{\frac{3}{2}}}\sqrt{\tau} n\omega\mu$,

(iii) $\|ds_-\|\|dx^c\| \leq \frac{\omega^3 n^3 \mu}{2\sqrt{(1-\beta)\tau}}$,

(iv) $\|dx_-\|\|ds^c\| \leq \frac{\omega^3 n^3 \mu}{2\sqrt{(1-\beta)\tau}}$,

(v) $\|ds_+\|\|dx^c\| \leq \frac{\beta(2-\beta)}{2(1-\beta)^2}\omega^2 n^2 \mu$,

(vi) $\|ds^c\|\|dx_+\| \leq \frac{\beta(2-\beta)}{2(1-\beta)^2}\omega^2 n^2 \mu$.

Given $(x, y, s) \in \mathcal{N}(\tau, \beta)$, the following lemma ensures that the algorithm reduces the duality gap for a specific choice of parameters.

Lemma 8. *Suppose $(x, y, s) \in \mathcal{N}(\tau, \beta)$. If $\alpha_1 = \sqrt{\frac{\beta\tau}{2n}}\alpha_2, \alpha_2 \leq \frac{1}{\sqrt{n}\omega^{\frac{3}{2}}}, \beta \leq \frac{1}{78}$ and $\tau \leq \frac{51}{100}$, then $\mu(\alpha) \leq \left(1 - \frac{\alpha_1}{250}\right)\mu$.*

Proof. By (19), Lemma 2 and the Cauchy-Schwartz inequality, we can find

$$\begin{aligned}
\mu(\alpha) &\leq \frac{1}{n}\Big(n\mu - \frac{\alpha_1}{2}(1-\tau)n\mu + \alpha_2\frac{2-\beta}{1-\beta}\sqrt{n}\beta\tau\mu \\
&\quad + \frac{\alpha_1^3}{2}(\|ds_-\|\|dx^c\| + \|dx_-\|\|ds^c\|) + \frac{\alpha_1\alpha_2}{2}(\|ds_+\|\|dx_-\| + \|ds_-\|\|dx_+\|)\Big) \\
&\leq \mu - \frac{(1-\tau)\alpha_1\mu}{2} + \frac{\alpha_1\sqrt{2\beta\tau}(2-\beta)\mu}{1-\beta} + \frac{\alpha_1^3\omega^3 n^2\mu}{2\sqrt{(1-\beta)\tau}} + \frac{\alpha_1\alpha_2\beta(2-\beta)\omega\sqrt{\tau}\mu}{(1-\beta)^{\frac{3}{2}}} \\
&\leq \left(1 - \Big(\frac{1-\tau}{2} - \frac{\sqrt{2\beta\tau}(2-\beta)}{1-\beta} - \frac{\beta\sqrt{\tau}}{4\sqrt{1-\beta}} - \frac{\beta(2-\beta)\sqrt{\tau}}{(1-\beta)^{\frac{3}{2}}\sqrt{n\omega}}\Big)\alpha_1\right)\mu \\
&\leq \left(1 - \left(\frac{49}{200} - \frac{155\sqrt{51}}{770\sqrt{39}} - \frac{\sqrt{3978}}{3120\sqrt{77}} - \frac{12090\sqrt{3978}}{4684680\sqrt{462}}\right)\alpha_1\right)\mu \leq \left(1 - \frac{\alpha_1}{250}\right)\mu,
\end{aligned}$$

where the second inequality follows from Corollary 2, the third one from $\alpha_1 = \sqrt{\frac{\beta\tau}{2n}}\alpha_2$ and $\alpha_2 \leq \frac{1}{\sqrt{n}\omega^{\frac{3}{2}}}$ and the fourth one is due to $\omega \geq 6, n \geq 1, \tau \leq \frac{51}{100}$ and $\beta \leq \frac{1}{78}$. Thus the proof is completed. □

Lemma 9. *Let* $\alpha_1 = \sqrt{\frac{\beta\tau}{2n}}\alpha_2,\ \alpha_2 \leq \frac{1}{\sqrt{n}\omega^{\frac{3}{2}}},\ \beta \leq \frac{1}{78},\ \tau = \frac{51}{100}$ *and* $(x, y, s) \in \mathcal{N}(\tau, \beta)$. *Then*

$$x(\alpha)^T s(\alpha) \geq \Big(1 - \frac{\alpha_1}{2}\Big)x^T s.$$

Proof. First, by using the inequalities $1 \leq \frac{\tau\mu e + v^2}{v^2} \leq 2$ and $\alpha_1 \leq \alpha_2$, we have

$$\begin{aligned}
\frac{\alpha_1}{2}e^T\left(\frac{\tau\mu e + v^2}{v^2}(\tau\mu e - v^2)^-\right) &+ \alpha_2 e^T\left(\frac{\tau\mu e + v^2}{v^2}(\tau\mu e - v^2)^+\right) \\
&\geq \frac{\alpha_1}{2}e^T\big(2(\tau\mu e - v^2)^-\big) + \alpha_2 e^T\big((\tau\mu e - v^2)^+\big) \\
&\geq \frac{\alpha_1}{2}e^T\big(2(\tau\mu e - v^2)^-\big) + \alpha_1 e^T\big((\tau\mu e - v^2)^+\big) \\
&= \alpha_1 e^T\big((\tau\mu e - v^2)\big) = \alpha_1(\tau - 1)n\mu.
\end{aligned}$$

Substituting this into (19), also using the Cauchy-Schwartz inequality, we obtain

$$\begin{aligned}
x(\alpha)^T s(\alpha) &\geq n\mu + \alpha_1(\tau - 1)n\mu - \frac{\alpha_1^3}{2}\big(\|ds_-\|\|dx^c\| + \|dx_-\|\|ds^c\|\big) \\
&\qquad - \frac{\alpha_1\alpha_2}{2}\big(\|ds_+\|\|dx_-\| + \|dx_+\|\|ds_-\|\big) \\
&\geq n\mu - \alpha_1(1-\tau)n\mu - \alpha_1^3\Big(\frac{\omega^3 n^3\mu}{2\sqrt{(1-\beta)\tau}}\Big) - \alpha_1\alpha_2\Big(\frac{\beta(2-\beta)}{(1-\beta)^{\frac{3}{2}}}\omega\sqrt{\tau}n\mu\Big) \\
&\geq \left(1 - \left(1 - \tau + \alpha_1^2\Big(\frac{\omega^3 n^2}{2\sqrt{(1-\beta)\tau}}\Big) + \alpha_2\Big(\frac{\beta(2-\beta)}{(1-\beta)^{\frac{3}{2}}}\omega\sqrt{\tau}\Big)\right)\alpha_1\right)n\mu \\
&\geq \left(1 - \Big(1 - \tau + \frac{\beta\sqrt{\tau}}{4\sqrt{1-\beta}} + \frac{\beta(2-\beta)\sqrt{\tau}}{(1-\beta)^{\frac{3}{2}}\sqrt{n\omega}}\Big)\alpha_1\right)n\mu \\
&\geq \left(1 - \Big(\frac{49}{100} + \frac{\sqrt{3978}}{3120\sqrt{77}} + \frac{12090\sqrt{3978}}{4684680\sqrt{462}}\Big)\alpha_1\right)n\mu \\
&= (1 - 0.4999\alpha_1)n\mu \geq \Big(1 - \frac{\alpha_1}{2}\Big)x^T s
\end{aligned}$$

Thus we have completed the proof. □

The proof of next lemma is quite similar to the proof of Lemma 6 in [13], so the proof is omitted here.

Lemma 10. *Suppose that $\mu(\alpha) > 0$ and $(x, y, s) \in \mathcal{N}(\tau, \beta)$, then we have*

$$\left\|(\tau\mu(\alpha)e - \Gamma(\alpha))^+\right\| \leq \big(1 - 2\alpha_2\big)\beta\tau\mu(\alpha),$$

where

$$\Gamma(\alpha) = v^2 + \frac{\tau\mu e + v^2}{v^2}\Big(\frac{\alpha_1}{2}(\tau\mu e - v^2)^- + \alpha_2(\tau\mu e - v^2)^+\Big).$$

Lemma 11. *If $\alpha = (\alpha_1, \alpha_2), \alpha_1 = \sqrt{\frac{\beta\tau}{2n}}\alpha_2, \alpha_2 \leq \frac{1}{\omega^{\frac{3}{2}} n^{\frac{3}{4}}}, \beta \leq \frac{1}{78}, \tau = \frac{51}{100}$ and $(x, y, s) \in \mathcal{N}(\tau, \beta)$, then*

$$(x(\alpha), y(\alpha), s(\alpha)) \in \mathcal{N}(\tau, \beta)$$

.

Proof. From (9), Lemma 10 and Corollary 2 we deduce that

$$\begin{aligned}
\left\|(\tau\mu(\alpha)e - x(\alpha)s(\alpha))^+\right\| &= \Big\|[\tau\mu(\alpha)e - \Gamma(\alpha) - \frac{\alpha_1\alpha_2}{2}(dx_+ds_- + ds_+dx_-) \\
&\quad - \frac{\alpha_1^3}{2}(ds_-dx^c + dx_-ds^c) - \alpha_2\alpha_1^2(ds_+dx^c + ds^c dx_+) \\
&\quad - \alpha_2^2(dx_+ds_+) - \alpha_1^4(dx^c ds^c)]^+\Big\| \\
&\leq \left\|[\tau\mu(\alpha)e - \Gamma(\alpha)]^+\right\| + \frac{\alpha_1\alpha_2}{2}(\|(dx_+\|\|ds_-\| + \|ds_+\|\|dx_-\|) \\
&\quad + \frac{\alpha_1^3}{2}(\|ds_-\|\|dx^c\| + \|dx_-\|\|ds^c\|) + \alpha_2\alpha_1^2(\|ds_+\|\|dx^c\| + \|dx_+\|\|ds^c\|) \\
&\quad + \alpha_2^2(\|dx_+\|\|ds_+\|) + \alpha_1^4(\|dx^c\|\|ds^c\|) \\
&\leq (1 - 2\alpha_2)\beta\tau\mu(\alpha) + \sqrt{\frac{\beta\tau}{2n}}\alpha_2^2\frac{\beta(2-\beta)}{(1-\beta)^{\frac{3}{2}}}\sqrt{\tau}\; n\omega\mu \\
&\quad + \Big(\frac{\beta\tau}{2n}\Big)^{\frac{3}{2}}\frac{\alpha_2^3\omega^3 n^3\mu}{2\sqrt{(1-\beta)\tau}} + \Big(\frac{\beta\tau}{2n}\Big)\frac{\alpha_2^3\beta(2-\beta)}{(1-\beta)^2}\omega^2 n^2\mu \\
&\quad + \frac{\alpha_2^2\beta^2(2-\beta)^2\tau\mu}{2(1-\beta)^3} + \Big(\frac{\beta^2\tau^2}{4n^2}\Big)\frac{\alpha_2^4\omega^4 n^4\mu}{8(1-\beta)\tau} \\
&\leq (1 - 2\alpha_2)\beta\tau\mu(\alpha) + \alpha_2\beta\tau\mu\Big(\frac{(2-\beta)\sqrt{\beta}}{n^{\frac{1}{4}}(1-\beta)^{\frac{3}{2}}\sqrt{2\omega}} + \frac{\sqrt{\beta}}{4\sqrt{2(1-\beta)}} \\
&\quad + \frac{\beta(2-\beta)}{2\sqrt{n}\omega(1-\beta)^2} + \frac{\beta(2-\beta)^2}{2\omega^{\frac{3}{2}} n^{\frac{3}{4}}(1-\beta)^3} + \frac{\beta}{32(1-\beta)\sqrt{\omega}n^{\frac{1}{4}}}\Big) \\
&\leq (1 - 2\alpha_2)\beta\tau\mu(\alpha) \\
&\quad + \alpha_2\beta\tau\mu\Big(\frac{155}{154\sqrt{231}} + \frac{1}{4\sqrt{154}} + \frac{155}{71148} + \frac{(155)^2}{(77)^3 \times 12\sqrt{6}} + \frac{1}{77 \times 32\sqrt{6}}\Big) \\
&\leq (1 - 2\alpha_2)\beta\tau\mu(\alpha) + 0.091\alpha_2\beta\tau\mu.
\end{aligned}$$

On the other hand, from lemma 9, $\alpha_1 = \sqrt{\frac{\beta\tau}{2n}}\alpha_2$ and $\alpha_2 \leq \frac{1}{n^{\frac{3}{4}}\omega^{\frac{3}{2}}}$ it follows that

$$\mu(\alpha) \geq \Big(1 - \frac{\alpha_1}{2}\Big)\mu \geq \Big(1 - \frac{\sqrt{\beta\tau}}{2\sqrt{2}n^{\frac{5}{4}}\omega^{\frac{3}{2}}}\Big)\mu.$$

Therefore, we can write

$$\begin{aligned}\big\|(\tau\mu(\alpha)e - x(\alpha)s(\alpha))^+\big\| - \beta\tau\mu(\alpha) &\leq -2\alpha_2\beta\tau\mu(\alpha) + 0.091\alpha_2\beta\tau\mu \\ &\leq -2\alpha_2\beta\tau\Big(1 - \frac{\sqrt{\beta\tau}}{2\sqrt{2}n^{\frac{5}{4}}\omega^{\frac{3}{2}}}\Big)\mu + 0.091\alpha_2\beta\tau\mu \\ &= \big(-2 + 0.002 + 0.091\big)\alpha_2\beta\tau\mu \leq 0.\end{aligned}$$

The proof is completed. □

Now, we state our main result concerning upper bound for the number of iterations in which Algorithm 1 stops with an ε-approximate solution.

Theorem 1. *Suppose that $\tau = \frac{51}{100}$ and $\beta \leq \frac{1}{78}$ are fixed for all iterations. Then Algorithm 1 will terminate in $\mathcal{O}\big(n^{\frac{5}{4}}\log\varepsilon^{-1}\big)$ iterations with a solution (x, y, s) for which $\mu \leq \varepsilon\mu_0$, $\|b - Ax\| \leq \varepsilon\|b - Ax^0\|$ and $\|c - s - A^Ty\| \leq \varepsilon\|c - s^0 - A^Ty^0\|$.*

Proof. By Lemma 8, at each iteration, we have

$$\mu_k \leq \Big(1 - \frac{\alpha_1}{250}\Big)\mu_{k-1} = \Big(1 - \frac{\alpha_1}{250}\Big)^k\mu_0 \leq \Big(1 - \frac{\sqrt{\beta\tau}}{250\sqrt{2}n^{\frac{5}{4}}\omega^{\frac{3}{2}}}\Big)^k\mu_0.$$

The above inequality implies that $\mu \leq \varepsilon\mu_0$ if $k \geq \frac{250\sqrt{2}\omega^{\frac{3}{2}}n^{\frac{5}{4}}}{\sqrt{\beta\tau}}\log\varepsilon^{-1}$. Thus the proof is completed. □

5. Conclusions

We have presented a predictor-corrector infeasible interior-point algorithm for LO based on a wide neighborhood of the central path. The predictor search direction induced by the kernel function is decomposed into two orthogonal different directions. The algorithm produces a linear combination of the composed predictor directions along with a second-order corrector step in the desired neighborhood. By using an elegant analysis, we proved that the iteration complexity of the algorithm is $\mathcal{O}(n^{\frac{5}{4}}\log\varepsilon^{-1})$.

References

[1] W. Ai, *Neighborhood-following algorithms for linear programming*, Science in China Series A: Mathematics **47** (2004), no. 6, 812–820.

[2] W. Ai and S. Zhang, *An $O(\sqrt{n}l)$ iteration primal-dual path-following method, based on wide neighborhoods and large updates, for monotone LCP*, SIAM J. Optim. **16** (2005), no. 2, 400–417.

[3] Y.-Q. Bai, M. El Ghami, and C. Roos, *A comparative study of kernel functions for primal-dual interior-point algorithms in linear optimization*, SIAM J. Optim. **15** (2004), no. 1, 101–128.

[4] Zs. Darvay, *A new predictor-corrector algorithm for linear programming*, Alkalmazott Matematikai Lapok (in Hungarian) **22** (2005), 135–161.

[5] Z. Feng, *A new iteration large-update primal-dual interior-point method for second-order cone programming*, Numer. Func. Anal. Optim. **33** (2012), no. 4, 397–414.

[6] Z. Feng and L. Fang, *A new $O(\sqrt{n}l)$ -iteration predictor-corrector algorithm with wide neighborhood for semidefinite programming*, J. Comput. Appl. Math. **256** (2014), 65–76.

[7] N.K. Karmarkar, *A new polynomial-time algorithm for linear programming*, Combinatorica **4** (1984), no. 4, 373–395.

[8] B. Kheirfam, *A predictor-corrector interior-point algorithm for $P_*(\kappa)$-horizontal linear complementarity problem*, Numer. Algorithms **66** (2014), no. 2, 349–361.

[9] ———, *A corrector–predictor path-following method for convex quadratic symmetric cone optimization*, J. Optim. Theory Appl. **164** (2015), no. 1, 246–260.

[10] ———, *A corrector–predictor path-following method for second-order cone optimization*, Int. J. Comput. Math. **93** (2016), no. 12, 2064–2078.

[11] ———, *A predictor-corrector infeasible-interior-point algorithm for semidefinite optimization in a wide neighborhood*, Fundam. Inform. **152** (2017), no. 1, 33–50.

[12] B. Kheirfam and M. Chitsaz, *A new second-order corrector interior-point algorithm for $P_*(\kappa)$-LCP*, Filomat **31** (2017), no. 20, 6379–6391.

[13] B. Kheirfam and M. Haghighi, *A wide neighborhood interior-point algorithm for linear optimization based on a specific kernel function*, Period. Math. Hung. **79** (2019), no. 1, 94–105.

[14] B. Kheirfam and M. Mohamadi-Sangachin, *A wide neighborhood second-order predictor-corrector interior-point algorithm for semidefinite optimization with modified corrector directions*, Fundam. Inform. **153** (2017), no. 4, 327–346.

[15] Y. Li and T. Terlaky, *A new class of large neighborhood path-following interior point algorithms for semidefinite optimization with $O\big(\sqrt{n}\log(\frac{tr(x^0s^0)}{\varepsilon})\big)$ iteration complexity*, SIAM J. Optim. **20** (2010), no. 6, 2853–2875.

[16] C. Liu and H. Liu, *A new second-order corrector interior-point algorithm for semidefinite programming*, Math. Meth. Oper. Res. **75** (2012), no. 2, 165–183.

[17] H. Liu, X. Yang, and C. Liu, *A new wide neighborhood primal–dual infeasible-interior-point method for symmetric cone programming*, J. Optim. Theory Appl. **158** (2013), no. 3, 796–815.

[18] N. Megiddo, *Pathways to the optimal set in linear programming*, Progress in Mathematical Programming, Springer-Verlag, New York, 1989, pp. 131–158.

[19] S. Mehrotra, *On the implementation of a primal-dual interior point method*, SIAM J. Optim. **2** (1992), no. 4, 575–601.

[20] S. Mizuno, M.J. Todd, and Y. Ye, *On adaptive-step primal-dual interior-point algorithms for linear programming*, Math. Oper. Res. **18** (1993), no. 4, 964–981.

[21] J. Peng, C. Roos, and T. Terlaky, *Self-regular functions and new search directions for linear and semidefinite optimization*, Math. Program. **93** (2002), no. 1, 129–171.

[22] F.A. Potra, *Interior point methods for sufficient horizontal LCP in a wide neighborhood of the central path with best known iteration complexity*, SIAM J. Optim. **24** (2014), no. 1, 1–28.

[23] C. Roos, T. Terlaky, and J.-P. Vial, *Theory and algorithms for linear optimization: an interior point approach*, John Wiley & Sons, Chichester, UK, 1997.

[24] G. Sonnevend, *An "analytic center" for polyhedrons and new classes of global algorithms for linear (smooth, convex) optimization*, In A. Prékopa and J. Szelezsán and B. Strazicky editor, System Modelling and Optimization: Proceedings of the 12th IFIP-Conference held in Budapest, Hungary, September 1985, Lecture Notes in Control and Information Sciences, 84, Springer Verlag, Berlin, West-Germany, 1986, pp. 866–876.

[25] G.-Q. Wang and Y.-Q. Bai, *A new full nesterov–todd step primal–dual path-following interior-point algorithm for symmetric optimization*, J. Optim. Theory Appl. **154** (2012), no. 3, 966–985.

[26] G.Q. Wang, Y.Q. Bai, X.Y. Gao, and D.Z. Wang, *Improved complexity analysis of full Nesterov–Todd step interior-point methods for semidefinite optimization*, J. Optim. Theory Appl. **165** (2015), no. 1, 242–262.

[27] G.Q. Wang, X.J. Fan, D.T. Zhu, and D.Z. Wang, *New complexity analysis of a full-newton step feasible interior-point algorithm for $P_*(\kappa)$-LCP*, Optim. Lett. **9** (2015), no. 6, 1105–1119.

[28] S.J. Wright, *Primal-Dual Interior-Point Methods*, SIAM, 1997.

[29] Y. Ye, *Interior Point Algorithms: Theory and Analysis*, vol. 44, John Wiley & Sons Inc., New York, 1997.

[30] Y. Zhang and D. Zhang, *On polynomiality of the mehrotra-type predictor–corrector interior-point algorithms*, Math. Program. **68** (1995), no. 1, 303–318.