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Outer-weakly convex domination number of graphs

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Abstract: For a given simple graph $G = (V, E)$, a set $S \subseteq V$ is an outer-weakly convex dominating set if every vertex in $V \setminus S$ is adjacent to some vertex in S and $V \setminus S$ is a weakly convex set. The *outer-weakly convex domination number* of a graph G, denoted by $\widetilde{\gamma}_{wcon}(G)$, is the minimum cardinality of an outer-weakly convex dominating set of G. In this paper, we initiate the study of outer-weakly convex domination as a new variant of graph domination and we show the close relationship that exists between this novel parameter and other domination parameters of a graph. Furthermore, we obtain general bounds on $\widetilde{\gamma}_{wcon}(G)$ and, for some particular families of graphs, we obtain closed formulae.

Keywords: convex domination, weakly-convex domination, outer-connected domination, outer-convex domination, outer-weakly convex domination

AMS Subject classification: 05C69

1. Introduction

Let $G = (V(G), E(G))$ be a simple graph. A graph G is *connected* if there is at least one path that connects every two vertices $x, y \in V(G)$, otherwise, G is *disconnected*. For any two vertices u and v in a connected graph, the distance $d_G(u, v)$ between u and v is the length of the shortest path in G. A u-v path of length $d_G(u, v)$ is also referred to as u-v *geodesic*. The *closed interval* $I_G[u, v]$ consists of all those vertices

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lying on a *u*-*v* geodesic in G. For a subset S of vertices of G, the union of all sets $I_G[u, v]$ for $u, v \in S$ is denoted by $I_G[S]$. Hence $x \in I_G[S]$ if and only if x lies on some *u*-*v* geodesic, where $u, v \in S$. A set $S \subseteq V(G)$ is *convex* if $I_G[S] = S$. In other words, a set S is *convex* in G if, for every two vertices $u, v \in S$, the vertex set of every u - v *geodesic* is contained in S. Certainly, if G is connected graph, then $V(G)$ is convex. On the other hand, a set $C \subseteq V(G)$ is *weakly convex* of G if for every two vertices $u, v \in G$ there exists a $u - v$ geodesic whose vertices belong to C. Convexity in graphs was studied in [2, 11].

A subset S of a vertex set $V(G)$ is a *dominating set* of G if for every vertex $v \in V(G)\backslash S$, there exists a vertex $x \in S$ such that xv is an edge of G. The *domination number* $\gamma(G)$ of G is the smallest cardinality of a dominating set S of G. Dominating sets have several applications in a variety of fields, including communication and electrical networks, protection and location strategies, data structures and others. Two books by Haynes et al., [12, 13], provide a comprehensive treatment of the general results on domination in graphs.

A dominating set S which is also convex is called a *convex dominating set* of G. The *convex domination number* $\gamma_{con}(G)$ of G is the smallest cardinality of a convex dominating set of G. A convex dominating set of cardinality $\gamma_{con}(G)$ is called a γ_{con}-*set* of G. A dominating set of G which is weakly convex is called a weakly convex dominating set. The weakly convex domination number of G, denoted by $\gamma_{wcon}(G)$, is the smallest cardinality of a weakly convex dominating set of G. The weakly convex and convex domination numbers investigated in [9, 14]. A set S of vertices of a graph G is an outer-connected dominating set if every vertex not in S is adjacent to some vertex in S and the subgraph induced by $V(G) \setminus S$, denoted $\langle V(G)\backslash S\rangle$, is connected. The outer-connected domination number $\widetilde{\gamma}_c(G)$ is the minimum cardinality of the outer-connected dominating set S of a graph G. The concept of outer-connected domination in graphs was introduced by Cyman [3] and further studied by others in [1, 5, 10]. A set S of vertices of a graph G is an outer-convex dominating set if every vertex not in S is adjacent to some vertex in S and the subgraph induced by $V(G) \setminus S$, denoted $\langle V(G)\backslash S\rangle$, is convex. The outer-convex domination number $\widetilde{\gamma}_{con}(G)$ is the minimum cardinality of the outer-convex dominating set S of a graph G. The concept of outer-convex domination in graphs was introduced by Dayap and Enriquez in 2019 [8] and further studied in [4, 6, 7].

Motivated by the definition of outer-connected and outer-convex domination in graphs, we define a new domination parameter in graphs called *outer-weakly convex domination*. A set S of vertices of a graph G is an outer-weakly convex dominating set if every vertex not in S is adjacent to some vertex in S and a set $V(G) \setminus S$ is weakly convex. The *outer-weakly convex domination number* of G, denoted by $\widetilde{\gamma}_{wcon}(G)$, is the minimum cardinality of an outer-weakly convex dominating set of G. An outer-weakly convex dominating set of cardinality $\widetilde{\gamma}_{wcon}(G)$ will be called a $\widetilde{\gamma}_{wcon}$-*set*.

In this paper, we initiate the study of outer-weakly convex domination as a new variant of graph domination and we show the close relationship that exists between this novel parameter and other domination parameters of a graph. Furthermore, we obtain general bounds on $\widetilde{\gamma}_{wcon}(G)$ and, for some particular families of graphs, we obtain closed formulae.

2. Preliminaries

We will use the notation K_n, P_n, C_n and $K_{1,n}$ for complete graphs, path graphs, cycle graphs and stars of order n and $K_{m,n}$ for complete bipartite graphs. The proof of next three results are straightforward and therefore omitted.

Observation 1. For a complete subgraph F of a graph G with order n, $\widetilde{\gamma}_{wcon}(G) \leq n - |V(F)| + 1$.

Observation 2.
1. For $n \geq 2$, $\widetilde{\gamma}_{wcon}(K_n) = 1$.
2. For $n \geq 4$, $\widetilde{\gamma}_{wcon}(P_n) = n - 2$.
3. For $n \geq 3$, $\widetilde{\gamma}_{wcon}(C_n) = n - 2$.
4. For $n \geq 1$, $\widetilde{\gamma}_{wcon}(K_{1,n}) = n - 1$.
5. For $n \geq m \geq 2$, $\widetilde{\gamma}_{wcon}(K_{m,n}) = 2$.

Observation 3. If G is a connected graph of order $n \geq 3$ with $\widetilde{\gamma}_{wcon}(G) \leq n - 2$, then any $\widetilde{\gamma}_{wcon}(G)$-set contains all leaves of G.

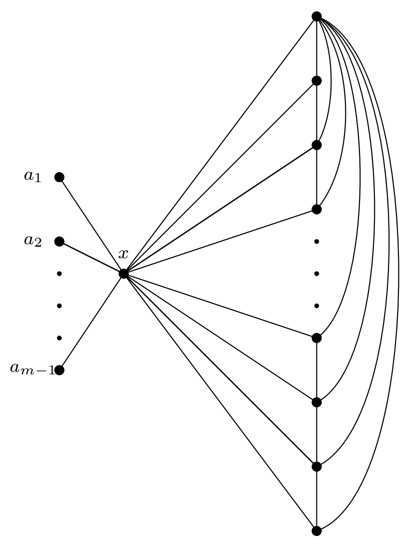

Figure 1. A graph G of order n and $\widetilde{\gamma}_{wcon}(G) = m$.

Theorem 4. Given positive integers m and n where $n \geq 2$ and $1 \leq m \leq n-1$, there exists a connected graph G of order n with $\widetilde{\gamma}_{wcon}(G) = m$.

Proof. If $m = 1$, then let $G = K_n$ and if $m = n-1$, then let $G = K_{1,n-1}$. Assume that $1 \leq m \leq n-2$. Let H^* be the graph obtained from a path P_{n-m} by joining one of its leaves to other vertices and let H be obtained from H^* by adding a new vertex x and joining x to all vertices of H and let G be obtained from H by attaching $m-1$ pendant edges at x (see Figure 1). Clearly the set $\{x, a_1, \dots, a_{m-1}\}$ is an outer-weakly convex dominating set implying that $\widetilde{\gamma}_{wcon}(G) \leq m$. To prove the inverse inequality, let S be a $\widetilde{\gamma}_{wcon}(G)$-set. Since $\widetilde{\gamma}_{wcon}(G) \leq n-2$, we deduce from Observation 3 that $\{a_1, \dots, a_{m-1}\} \subseteq S$. On the other hand, to dominate the vertices of H^* we must have $|S \cap V(H)| \geq 1$ yielding $\widetilde{\gamma}_{wcon}(G) \geq m$. Thus $\widetilde{\gamma}_{wcon}(G) = m$ and the proof is complete. □

Here, we investigate the relations between the outer-weakly convex domination number and other types of domination numbers such as outer-connected and outer-convex domination number.

Theorem 5. For any graph G, $\widetilde{\gamma}_c(G) \leq \widetilde{\gamma}_{wcon}(G)$ and $\widetilde{\gamma}_{wcon}(G) \leq \widetilde{\gamma}_{con}(G)$.

Proof. Since every outer-weakly convex dominating set of G is an outer-connected dominating set of G, we have $\widetilde{\gamma}_c(G) \leq \widetilde{\gamma}_{wcon}(G)$. Similarly, since every outer-convex dominating set of G is an outer-weakly connected dominating set of G, we have $\widetilde{\gamma}_{wcon}(G) \leq \widetilde{\gamma}_{con}(G)$. □

By Theorem 5, we have following inequality chain.

Corollary 1. *Let G be a nontrivial connected graph. Then*

$$\gamma(G) \leq \widetilde{\gamma}_c(G) \leq \widetilde{\gamma}_{wcon}(G) \leq \widetilde{\gamma}_{con}(G).$$

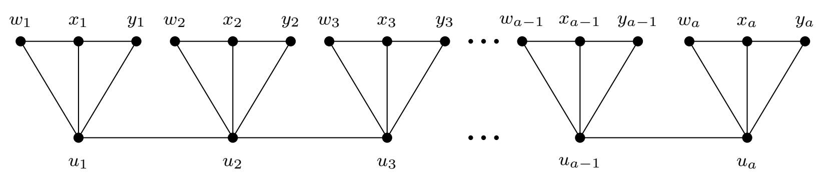

Figure 2. A graph G of order $4a$ with $\widetilde{\gamma}_c(G) = \widetilde{\gamma}_{wcon}(G) = a$ and $\widetilde{\gamma}_{con}(G) = 2a$.

Theorem 6. For any integer $a \geq 1$, there exists a connected graph G of order $4a$ such that $\widetilde{\gamma}_c(G) = \widetilde{\gamma}_{wcon}(G) = a$ and $\widetilde{\gamma}_{con}(G) = 2a$.

Proof. Let G be a graph obtained from a path $P = u_1u_2 \ldots u_a$ and a copies of P_3 by joining u_i to every vertex in the i-th copy of P_3. Assume that $w_ix_iy_i$ is the i-th copy of P_3 (see Figure 2).

We first show that $\widetilde{\gamma}_c(G) = a$. Let S be a $\widetilde{\gamma}_c(G)$-set. To dominate y_i we must have $|S \cap \{u_i, w_i, x_i, z_j\}| \geq 1$ for each i, and so $\widetilde{\gamma}_c(G) = |S| \geq n$. On the other hand, it is easy to see that the set $\{y_1, \ldots, y_a\}$ is an outer-connected dominating set of G implying that $\widetilde{\gamma}_c(G) \leq a$. Thus $\widetilde{\gamma}_c(G) = a$. Likewise, we have $\widetilde{\gamma}_{wcon}(G) = a$.

Next we show that $\widetilde{\gamma}_{con}(G) = 2a$. Clearly the set $\{x_i, y_i \mid 1 \leq i \leq a\}$ is an outer-convex dominating set of G and so $\widetilde{\gamma}_{con}(G) \leq 2a$. To prove the inverse inequality, let S be a $\widetilde{\gamma}_{con}(G)$-set. It is enough to show that $|V(G) - S| \leq 2a$. To this end, we show that $|(V(G) - S) \cap \{u_i, x_i, y_i, z_i\}| \leq 2$ for each i. Suppose, to the contrary, $|(V(G) - S) \cap \{u_i, x_i, y_i, z_i\}| \geq 3$ for some i. If $x_i, z_i \in V(G) - S$, then we must have $u_i, y_i \in S$ since $V(G)$ is a convex set and this leads to a contradiction because S is a dominating set. Assume without loss of generality that $z_i \notin V(G) - S$. By the assumption we have $x_i, y_i, u_i \in V(G) - S$ and this leads to a contradiction because x_i is not dominated by S. Therefore $|V(G) - S| \leq 2a$ yielding $\widetilde{\gamma}_{con}(G) = |S| \geq 2a$. Thus $\widetilde{\gamma}_{con}(G) = 2a$ and the proof is complete. □

3. Graphs with large and small outer-weakly convex domination number

Theorem 7. For any connected graph G of order $n \geq 2$,

$$1 \leq \widetilde{\gamma}_{wcon}(G) \leq n - 1.$$

The equality holds in upper bound if and only if G is a star, and the equality holds in lower bound if and only if G has a universal vertex v such that the diameter of the subgraph of G induced by $V(G) - \{v\}$ is at most two.

Proof. Let G be a nontrivial connected graph of order n. Obviously $\widetilde{\gamma}_{wcon}(G) \geq 1$. For the upper bound, we note that since the subgraph induced by a single vertex is weakly convex, $V(G) - \{u\}$ is an outer-weakly convex dominating set of G for each vertex $u \in V(G)$ and so $\widetilde{\gamma}_{wcon}(G) \leq n - 1$.

If G is a star, then by Observation 2 we have $\widetilde{\gamma}_{wcon}(G) = n - 1$. Conversely, let $\widetilde{\gamma}_{wcon}(G) = n - 1$. If G has a cycle $(x_1x_2 \ldots, x_kx_1)$, then clearly $V(G) - \{x_1, x_2\}$ is an outer-weakly convex dominating set of G which leads to a contradiction. Hence G is acyclic and so is a tree. If $\operatorname{diam}(G) \geq 3$ and $x_1x_2 \ldots x_k$ $(k \geq 4)$ is a diametral path in G, then clearly $V(G) - \{x_2x_3\}$ is an outer-weakly convex dominating set of G which leads to a contradiction again. Thus $\operatorname{diam}(G) \leq 2$ and so G is a star.

If G has a universal vertex v such that the diameter of the subgraph of G induced by $V(G) - \{v\}$ is at most two, then clearly $\{v\}$ is an outer-weakly convex dominating set of G and so $\widetilde{\gamma}_{wcon}(G) = 1$. Conversely, let $\widetilde{\gamma}_{wcon}(G) = 1$. Assume that $S = \{v\}$ is an outer-weakly convex dominating set of G. Since S is a dominating set and

$V(G)-\{v\}$ is a weakly convex set of G, we deduce that v is a universal vertex and that the subgraph of G induced by $V(G)-\{v\}$ is connected. If $\mathrm{diam}(G[V(G)-\{v\}]) \geq 3$ and u, w be two vertices in $V(G)-\{v\}$ with distance at least three in $G[V(G)-\{v\}]$, then any $u-v$ geodesic in Gpassing through v which leads to a contradiction. Thus $\mathrm{diam}(G[V(G)-\{v\}]) \leq 2$ and the proof is complete. □

Theorem 8. If G is a disconnected graph and $G_1, G_2, \ldots, G_s$ are the components of G, then

$$\widetilde{\gamma}_{wcon}(G) = n(G) - \max\{n(G_i) - \widetilde{\gamma}_{wcon}(G_i) \mid i = 1, 2, \ldots, s\}.$$

Proof. Let $S_1, S_2, \ldots, S_t$ be minimum outer-weakly convex dominating sets of $G_1, G_2, \ldots, G_t$, respectively. Clearly $V(G) \setminus (V(G_i) \setminus S_i)$ is an outer-weakly convex dominating set of G for each i, and so

$$\begin{aligned}\widetilde{\gamma}_{wcon}(G) &\leq \min\{n(G) - (n(G_i) - \widetilde{\gamma}_{wcon}(G_i)) \mid i = 1, 2, \ldots, t\} \\ &= n(G) - \max\{n(G_i) - \widetilde{\gamma}_{wcon}(G_i) \mid i = 1, 2, \ldots, t\}.\end{aligned}$$

Now, let S be a $\widetilde{\gamma}_{wcon}(G)$-set. Since $V(G) \setminus S$ is a weakly convex set, we must have $V(G) \setminus S \subseteq V(G_i)$ for some $i \in \{1, 2, \ldots, t\}$ and from the minimality of S it follows that $S \cap V(G_i)$ is a minimum outer-weakly convex dominating set of G_i. Hence, $S \cap V(G_i) = \widetilde{\gamma}_{wcon}(G_i)$ and we have

$$\begin{aligned}\widetilde{\gamma}_{wcon}(G) &= n(G) - (n(G_i) - \widetilde{\gamma}_{wcon}(G_i)) \\ &\geq n(G) - \max\{n(G_i) - \widetilde{\gamma}_{wcon}(G_i) \mid i = 1, 2, \ldots, t\}.\end{aligned}$$

Thus $\widetilde{\gamma}_{wcon}(G) = n(G) - \max\{n(G_i) - \widetilde{\gamma}_{wcon}(G_i) \mid i = 1, 2, \ldots, s\}$ and the proof is complete. □

The following corollary follows directly from Theorems 7 and 8.

Corollary 2. *If $G_1, G_2, \ldots, G_s$ are the components of a graph G, then*

$$\widetilde{\gamma}_{wcon}(G) \geq \widetilde{\gamma}_{wcon}(G_1) + \widetilde{\gamma}_{wcon}(G_2) + \cdots + \widetilde{\gamma}_{wcon}(G_s).$$

The equality holds if and only if $s-1$ components of G are K_1.

Let $C = (x_1x_2x_3x_1)$ be a cycle of length 3. Assume B_p is the graph obtained from C by adding $p \geq 1$ pendant edges at x_1 and $B_{p,q}$ is the graph obtained from C by adding $p \geq 1$ pendant edges at some x_1 and $q \geq 1$ pendant edges at some x_2. Let B^p is the graph obtained from $K_{1,s}$ with $s \geq 3$, by subdividing one pendent edge $t \geq 1$ times. Assume $B^{p,q}$ is the graph obtained from $DS(p,q)$ with $p \geq 2$ and $q \geq 2$, by subdividing the non-pendant edge $t \geq 1$ times.

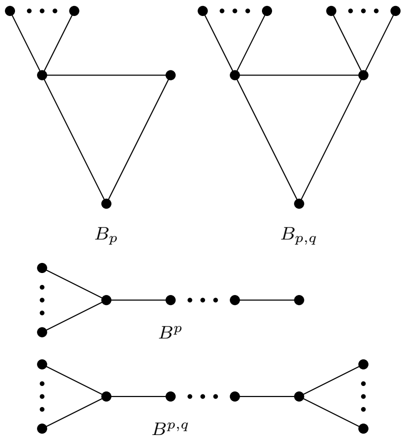

Figure 3. The graphs B_p, $B_{p,q}$, B^p and $B^{p,q}$.

Theorem 9. *Let G be a connected graph of order $n \geq 2$. Then, $\widetilde{\gamma}_{wcon}(G) = n - 2$ if and only if one of the following holds.*

(a) G is a path or a cycle.

(b) G is isomorphic to B_p or $B_{p,q}$.

(c) G is isomorphic to B^p or $B^{p,q}$.

Proof. Let G satisfy conditions $(a), (b)$ or (c). If G satisfies condition (a), then by Observation 2, we have $\widetilde{\gamma}_{wcon}(G) = n - 2$. Let G satisfy condition (b). Theorem 7 implies that $\widetilde{\gamma}_{wcon}(G) \leq n - 2$. Now we show that $\widetilde{\gamma}_{wcon}(G) \geq n - 2$. Let S be $\widetilde{\gamma}_{wcon}(G)$-set. If G has a leaf u such that $u \in V(G) - S$, then we must have $S = V(G) - \{u\}$, since $V(G) - S$ is a weakly convex set, a contradiction with $\widetilde{\gamma}_{wcon}(G) \leq n - 2$. Thus all leaves of G belongs to S. On the other hand, since one of the vertices of $\{x_1, x_2, x_3\}$, say x_3, has degree 2, to dominate x_3 we must have $S \cap \{x_1, x_2, x_3\} \neq \emptyset$. Hence $\widetilde{\gamma}_{wcon}(G) = |S| \geq n - 2$ and so $\widetilde{\gamma}_{wcon}(G) = n - 2$. Likewise, if G satisfies condition (c), then we can see $\widetilde{\gamma}_{wcon}(G) = n - 2$.

Conversely, let $\widetilde{\gamma}_{wcon}(G) = n - 2$. If G is a cycle or a path, we are done. Assume that G is neither a cycle nor a path.

First let G has a cycle $C = (x_1 x_2 \ldots x_m x_1)$. Since G is not a cycle, we may assume that $\deg(x_1) \geq 3$. If $m \geq 4$, then $V(G) - \{x_1, x_2, x_m\}$ is an outer-weakly convex dominating set of G of size $n - 3$ which is a contradiction. Hence $m = 3$. If $\deg(x_i) \geq 3$ for each $i = 1, 2, 3$, then $V(G) - \{x_1, x_2, x_3\}$ is an outer-weakly convex dominating set of G of size $n - 3$, a contradiction again. Hence, we may assume without loss of generality that $\deg(x_3) = 2$. If there is a path $x_i y_1 y_2$ in G such that $y_1, y_2 \notin \{x_1, x_2, x_3\}$, then the set $V(G) - \{y_1, x_i, x_{i+1}\}$ is an outer-weakly convex dominating set of G of size $n - 3$, a contradiction again. Suppose there is no path $x_i y_1 y_2$ in G where $y_1, y_2 \notin \{x_1, x_2, x_3\}$. If there is a vertex $y \in V(G) - \{x_1, x_2, x_3\}$ which is adjacent to some x_1, x_2, then

clearly $V(G) - \{y, x_2, x_3\}$ is an outer-weakly convex dominating set of G of size $n-3$ which is a contradiction again. Thus each vertex in $V(G) - \{x_1, x_2, x_3\}$ is a leaf adjacent to either x_1 or x_2 and so G is isomorphic to B_p or $B_{p,q}$.
Now let G be tree. Since $\widetilde{\gamma}_{wcon}(G) = n-2$, we deduce from Theorem 7 that $\text{diam}(G) \geq 3$. Let $x_1x_2\ldots,x_k$ $(k \geq 4)$ be a diametral path in G. If $\deg(x_i) \geq 3$ for some $3 \leq i \leq k-2$, then $V(G)-\{x_{i-1}, x_i, x_{i+1}\}$ is an outer-weakly convex dominating set of G of size $n-3$ which is a contradiction. Thus $\deg(x_i) = 2$ for each $3 \leq i \leq k-2$. Since G is not a path, we may assume that $\deg(x_2) \geq 3$. If $\deg(x_{k-1}) = 2$, then $G \cong B^p$ where $p = \deg(x_2) - 2$, and if $\deg(x_{k-1}) \geq 3$, then $G \cong B^{p,q}$ where $p = \deg(x_2) - 2$ and $q = \deg(x_{k-1}) - 2$. This completes the proof. □

4. Conclusion

An outer-weakly convex dominating set is a new variant of domination in graphs. Hence, this paper is a contribution to the development of domination theory in general. Since this is new, further investigations must be promoted to come up with coherent and substantial results of the parameter, an outer-weakly convex domination number. Thus, binary operations of graphs such as, the join, the sequential join, the corona, the lexicographic, and the Cartesian product of two graphs of an outer-weakly convex dominating sets are recommended for further study. Moreover, the corresponding bounds of the binary operations of two graphs are further look into. Finally, domination in graphs is rich with immediate applications in the real world such as routing problems in the internet, problems in electrical networks, data structures, neural and communication networks, data security, location strategies, and many others. The outer-weakly convex domination in graphs is not far from these applications.
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References

[1] M.H. Akhbari, R. Hasni, O. Favaron, H. Karami, and S.M. Sheikholeslami, *On the outer-connected domination in graphs*, J. Comb. Optim. **26** (2013), no. 1, 10–18.
[2] G. Chartrand and P. Zhang, *Convex sets in graphs*, Congr. Numer. **136** (1999), 19–32.
[3] J. Cyman, *The outer-connected domination number of a graph*, Australas. J. Combin. **38** (2007), 35–46.
[4] J.A. Dayap, *Outer-convex domination in the corona of graphs*, TWMS Journal of Applied and Engineering Mathematics, (to appear).

[5] J.A. Dayap, J.P. Dequillo, W.V. Rios, R.M.T. Telen, and A.Q. Sollano, *Securing chemical formula using polyalphabetic affine cipher*, Journal of Global Research in Mathematical Archives **6** (2019), no. 9, 6–14.

[6] J.A. Dayap, J.S. Dionsay, and R.T. Telen, *Perfect outer-convex domination in graphs*, International Journal of Latest Engineering Research and Applications **3** (2018), no. 7, 25–29.

[7] J.A. Dayap and E.L. Enriquez, *Outer-convex domination in the composition and cartesian product of graphs*, Journal of Global Research in Mathematical Archives **6** (2019), no. 3, 34–41.

[8] ———, *Outer-convex domination in graphs*, Discrete Math. Algorithms Appl. **12** (2020), no. 1, Article ID: 2050008.

[9] M. Dettlaff, M. Lemańska, S. Kosary, and S.M. Sheikholeslami, *The convex domination subdivision number of a graph*, Commun. Comb. Optim. **1** (2016), no. 1, 43–56.

[10] E. Enriquez, V. Fernandez, T. Punzalan, and J.A. Dayap, *Perfect outer-connected domination in the join and corona of graphs*, Recoletos Multidisciplinary Research Journal **4** (2016), no. 2, 1–8.

[11] F. Harary and J. Nieminen, *Convexity in graphs*, J. Differ Geom. **16** (1981), no. 2, 185–190.

[12] T.W. Haynes, S.T. Hedetniemi, and P. Slater, *Fundamentals of Domination in Graphs*, CRC press, 1998.

[13] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, *Domination in Graphs: Advanced Topics*, Marcel Dekker, New York, 1998.

[14] M. Lemańska, *Weakly convex and convex domination numbers*, Opuscula Math. **24** (2004), no. 2, 181–188.